Math Diversion 1088

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> In algebra problems, if you can arrange it, work
> with "nice" quantities. — The Author

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Source: https://www.youtube.com/watch?v=-fr70Fsq2iY
Title: Canada | Can You Solve This?
 Presenter: HkLogics
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1 Problem

Given the relation

$$\sqrt{2}^{\sqrt{x}} - \sqrt{2}^{\sqrt{y}} = 48\,, \tag{1}$$

find integer solutions.

2 Solution

Let's take the above advice and look for "nice" quantities. Set

$$2u = \sqrt{x}\,, \quad 2v = \sqrt{y}\,. \tag{2}$$

Then (1) becomes

$$2^u - 2^v = 2^4 \cdot 3\,. \tag{3}$$

After dividing through by 2^4, we have that

$$2^{u-4} - 2^{v-4} = 3\,. \tag{4}$$

Now, there are a couple things that have to jump out at us at this point. Whenever $a - b$ is positive, then $a > b$. Applying this principle to (4), we conclude that

$$2^{u-4} > 2^{v-4}\,. \tag{5}$$

This implies that a lower bound on 2^{u-4}, which forces $u-4 > 0$, so that 2^{u-4} is an even number. Okay, then what power of 2^{v-4} will produce an odd number? (We need to get an odd number on the RHS of (4).) So, we have only one choice for v, namely,

$$v = 4 \implies y = 64\,. \tag{6}$$

Using $v = 4$ in (4), we get

$$2^{u-4} - 2^0 = 3\,, \tag{7}$$

or

$$2^{u-4} = 4\,, \tag{8}$$

which forces $u = 6$, and thus

$$x = 144\,. \tag{9}$$