Math Diversion 1090

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> Genius is one percent inspiration and
> ninety-nine percent perspiration.
> — Thomas Edison

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Source: https://www.youtube.com/watch?v=BgtvGKsTOtM
Title: Harvard Entrance Exam Maths Question
 Presenter: Math Beast
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1 Problem

Given the relation

$$1 + z + z^2 + z^3 + z^4 + z^5 + z^6 = 0 \quad \text{where} \quad z \neq 1\,, \tag{1}$$

find the value of

$$\phi = z + z^2 + z^4\,. \tag{2}$$

2 Solution

On multiplying (1) through by $(1 - z)$, we have that (with some algebra)

$$1 - z^7 = 0\,. \tag{3}$$

In the complex numbers, we get seven roots:

$$z_k = e^{2\pi i k/7} \quad \text{where} \quad k \in [0..6]\,. \tag{4}$$

So, which root should we use for z in ϕ? I'll use the root for $k = 1$:

$$z = z_1 = e^{2\pi i/7}\,. \tag{5}$$

Then

$$\phi = e^{2\pi i/7} + e^{4\pi i/7} + e^{8\pi i/7}\,. \tag{6}$$

If you wish, you can expand each exponential by using Euler's formula, to get ϕ in terms of sines and cosines, but I won't bother.