Math Diversion 1094

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I love it when a plan comes together.
— Hannibal Smith, *The A-Team*

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Source:  https://www.youtube.com/watch?v=d4yF_duiq-c
Title:  Can You Pass Stanford University Entrance Exam?
 Presenter: Sagans Online Maths
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1 Problem

Given the relation

$$8^{\log x} - 2^{\log x} = 5! \,, \tag{1}$$

solve for all real values of x, where the logarithm is base 10.

2 Solution

There is a common theme in this type of problem: What's the relation between 8 and 2? It's $8 = 2^3$. So then (1) becomes

$$2^{3 \log x} - 2^{\log x} = 5! = 120 \,. \tag{2}$$

Now, for a change of variable. Let

$$y = 2^{\log x} \,, \tag{3}$$

then we get

$$y^3 - y - 120 = 0 \,. \tag{4}$$

On trying a few low-valued positive integers for y, we get one solution to (4) being

$$y = 5 \,. \tag{5}$$

On solving (3) for x, we have that

$$x = 10^{(\log 5)/(\log 2)} \,. \tag{6}$$

Now, since we know one root to (4), we can factor it to get

$$(y-5)(y^2+5y+24)=0\,. \tag{7}$$

On checking the roots to

$$y^2+5y+24=0\,, \tag{8}$$

we find that they are nonreal, so we dismiss them for this problem. And we are finished with (6). Though it can be re-expressed in the form

$$x=\left(10^{(\log 5)}\right)^{1/\log 2}=5^{1/\log 2}\,, \tag{9}$$

which is how WolframAlpha gives it.