Math Diversion 1102

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> Nothing that's really important is
> broken through very quickly.
> — George Martin

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Source: The Ether of Great Mathematical Ideas
Title: An Integral
 Presenter: Patrick
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1 Problem

Perform the indefinite integration:

$$I = \int \sin x \cos x \, dx \,. \tag{1}$$

2 Solution

$$I = \tfrac{1}{2} \int D_x (\sin x)^2 \, dx = \tfrac{1}{2} \int d(\sin x)^2 = \tfrac{1}{2} \int d(\sin^2 x) = \tfrac{1}{2} \sin^2 x + C \,. \tag{2}$$

But, we could also do the integration this way:

$$I' = -\tfrac{1}{2} \int D_x (\cos x)^2 \, dx = -\tfrac{1}{2} \int d(\cos x)^2 = -\tfrac{1}{2} \int d(\cos^2 x) = -\tfrac{1}{2} \cos^2 x + C' \,. \tag{3}$$

Can these both be correct? Yes, because

$$\sin^2 x + \cos^2 x = 1 \,. \tag{4}$$

Hence

$$I' = -\tfrac{1}{2} \cos^2 x + C' = -\tfrac{1}{2}(1 - \sin^2 x) + C' = \tfrac{1}{2} \sin^2 x + (C' - \tfrac{1}{2}) \,. \tag{5}$$

And this answer is the same as that of (2) when we identify $C = C' - \frac{1}{2}$.