

Math Diversion Problem 108

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You cannot ask us to take sides against arithmetic.
— Winston Churchill

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=IGUh3Mv7DAI>

Title: One Of The Most Difficult Harvard's University Entrance Exam

Presenter: Maths with Chinwendu

1 The Problem

Given the relation

$$x^x y^y = 8x^y y^x, \quad (1)$$

find the smallest positive integer values of x, y with $x > y$.

2 The Solution

Begin by dividing through by $x^y y^x$:

$$x^{x-y} y^{y-x} = 8. \quad (2)$$

On further simplification,

$$\left(\frac{x}{y}\right)^{x-y} = 8 = 2^3. \quad (3)$$

So, look: Since 8 is such a small number, let's just guess!

$$\frac{x}{y} = 2. \quad (4)$$

But $2^3 = 8$, so

$$x - y = 3. \quad (5)$$

On using this with (4), we get

$$x = 6, \quad y = 3. \quad (6)$$