

# Math Diversion Problem 111

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Mathematics knows no races or geographic boundaries;  
for mathematics, the cultural world is one country.  
— David Hilbert

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=uofFyT0d1Wc>  
Title: A Nice Algebra Equations  
Presenter: MathMinds

## 1 The Problem

Given the relations

$$x^2 + xy + x = 10, \quad (1a)$$

$$y^2 + xy + y = 20, \quad (1b)$$

find the values of  $x, y$ .

## 2 The Solution

By adding and subtracting the givens, we get

$$x^2 + y^2 + 2xy + x + y = 30, \quad (2a)$$

$$x^2 - y^2 + x - y = -10, \quad (2b)$$

These can be rewritten as

$$(x + y)^2 + x + y = 30, \quad (3a)$$

$$(x + y)(x - y) + x - y = -10. \quad (3b)$$

On setting

$$A \equiv x + y, \quad B = x - y, \quad (4)$$

these can be rewritten as

$$A^2 + A - 30 = 0, \tag{5a}$$

$$AB + B + 10 = 0, \tag{5b}$$

whose solutions are<sup>1</sup>

$$A_{\pm} = 5, -6, \tag{6a}$$

$$B_{\pm} = -5/3, 2. \tag{6b}$$

From (4), we have that

$$x = (A + B)/2, \tag{7a}$$

$$y = (A - B)/2. \tag{7b}$$

Therefore,

$$(x_+, y_+) = (5/3, 10/3), \tag{8a}$$

$$(x_-, y_-) = (-2, -4). \tag{8b}$$

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<sup>1</sup>Solve for  $A$  first.