

# Math Diversion Problem 113

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Learning is a treasure that will follow its owner everywhere.  
— Chinese proverb

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=ZQZMu0SraSQ>

Title: Can you Pass Harvard University Admission Interview

Presenter: Enjoy Math

## 1 The Problem

Given the relation

$$\sqrt{\frac{4^{20} - 2^{21} + 1}{2^{20} + 2^{11} + 1}} = 2^x - 1, \quad (1)$$

find the values of  $x$ .

## 2 The Solution

Let's begin by doing a trivial adjustment on the bases so that they are consistent:

$$\sqrt{\frac{2^{40} - 2 \cdot 2^{20} + 1}{2^{20} + 2 \cdot 2^{10} + 1}} = 2^x - 1, \quad (2)$$

Now we can do some factoring and then cancelling:

$$\sqrt{\frac{(2^{20} - 1)^2}{(2^{10} + 1)^2}} = \sqrt{\frac{(2^{10} + 1)^2 (2^{10} - 1)^2}{(2^{10} + 1)^2}} \quad (3a)$$

$$= (2^{10} - 1) \quad (3b)$$

$$= 2^x - 1. \quad (3c)$$

Hence  $x = 10$ .