

Math Diversion Problem 114

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Keep an open mind. That's the secret.
— Doctor Who

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=VgIImBmYKCE>
Title: Harvard University Entrance Exam
Presenter: SchoolClass Math

1 The Problem

Given the relation

$$m^2 - m^3 = 36, \tag{1}$$

find the values of m .

2 The Solution

Let's begin by converting (1) to standard form:

$$m^3 - m^2 + 36 = 0, \tag{2}$$

Now, we really should suspect there is a simple integer root to this (don't forget to try negative integers). After just a little effort, I found the -3 is a root by simply trying it in the equation. And after I applied long division to (2), I found that it factors as

$$(m + 3)(m^2 - 4m + 12) = 0, \tag{3}$$

And, according to the quadratic formula, the remaining quadratic factor has roots

$$m_{\pm} = 2 \pm 2i\sqrt{2}, \tag{4}$$