

# Math Diversion Problem 118

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The secret to perseverance is to just keep doing it.  
— The Author

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=wuM1d8B9d5k>  
Title: A Nice Math Olympiad Problem  
Presenter: Maths Black Board

## 1 The Problem

Given the relations

$$\sqrt{a} + \sqrt{2-b} = \sqrt{2}, \quad (1a)$$

$$\sqrt{b} + \sqrt{2-a} = \sqrt{2}, \quad (1b)$$

find the values of  $a, b$ .

## 2 The Solution

We begin by re-arranging both sides of both given relations, and then squaring:

$$a = 2 - 2\sqrt{2}\sqrt{2-b} + (2-b), \quad (2a)$$

$$b = 2 - 2\sqrt{2}\sqrt{2-a} + (2-a). \quad (2b)$$

Next, if we subtract the second of these from the first, we get (eventually)

$$\sqrt{2-b} = \sqrt{2-a}, \quad (3)$$

implying that

$$a = b. \quad (4)$$

Thus, (2a) can be rewritten as

$$a = 2 - 2\sqrt{2}\sqrt{2-a} + (2-a), \quad (5)$$

which becomes

$$a = 2 - \sqrt{2}\sqrt{2-a}, \quad (6)$$

which becomes with a bit of algebra

$$a(a-2) = 0. \quad (7)$$

Thus we have the solution pairs:

$$(a, b) = (0, 0), (2, 2). \quad (8)$$