

Math Diversions, Problem 13

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1 Problem

This problem is found on the YouTube channel **MindYourDecisions**, from April 17, 2021. My solution here is a little different from that given by the presenter:

<https://www.youtube.com/watch?v=3jnbBVp0f40>

Titled: A tricky problem with a “divine” answer!

Statement of the problem:

Solve for real values of x in the following equation:

$$\left(x - \frac{1}{x}\right)^{1/2} + \left(1 - \frac{1}{x}\right)^{1/2} = x. \quad (1a)$$

Solution:

It doesn't take long to realize that the LHS of (1a) is always going to be greater than zero, hence the RHS must also be greater than zero. Therefore, our solution x must be positive. Next, we define a new variable y as:

$$\left(x - \frac{1}{x}\right)^{1/2} - \left(1 - \frac{1}{x}\right)^{1/2} \equiv y. \quad (1b)$$

Next, we find the sum and product of x and y .

$$x + y = 2 \left(x - \frac{1}{x}\right)^{1/2}, \quad (2)$$

$$xy = x - 1. \quad (3)$$

From this last equation, we get

$$y = \frac{x-1}{x} = 1 - \frac{1}{x}. \quad (4)$$

On eliminating y between this last equation and (2), we have that

$$x + 1 - \frac{1}{x} = 2 \left(x - \frac{1}{x} \right)^{1/2}. \quad (5)$$

Now, when I inserted this last equation into Wolframalpha.com equation solver, I got back

$$x = \frac{1}{2} - \frac{\sqrt{5}}{2}, \quad x = \frac{1}{2} + \frac{\sqrt{5}}{2}. \quad (6)$$

Out of the two, I must choose the positive one:

$$x = \frac{1 + \sqrt{5}}{2}, \quad (7)$$

which is the Golden Ratio value.