

Math Diversion Problem 138

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Abstract

Here we use the unipodal algebra to assist in solving the problem, which is given to us on YouTube. Although I'm referring to the series under the name 'olympiad', the problems are from diverse sources as olympiads, entrance exams, SATs, and the like.

Don't ever take a fence down until you know the reason it was put up.

— Chesterton

The YouTube video is found at:

Source: https://www.youtube.com/watch?v=7zwAqqH6C58&list=PLMvuVe0n1Hd_KIT-dsvIVluQQN3pJrlmX&index=691
Title: Nice Algebra Math Simplification
Presenter: Master T Maths Class

1 The Problem

Given the relation

$$\left(\frac{x}{5}\right)^x = 5^{5^2}, \quad (1)$$

find the values of x over the real numbers.

2 The Solution

The first thing I did was to make my standard variable substitution:

$$x = 5^\alpha. \quad (2)$$

Then we get

$$\left(\frac{5^\alpha}{5}\right)^{5^\alpha} = 5^{5^2}, \quad (3)$$

which simplifies to

$$\left(5^{(\alpha-1)}\right)^{5^\alpha} = 5^{5^2}, \quad (4)$$

and then to

$$5^{5^\alpha(\alpha-1)} = 5^{5^2}. \quad (5)$$

On setting exponents equal, we have that

$$5^\alpha(\alpha - 1) = 5^2, \quad (6)$$

or

$$5^{\alpha-2}(\alpha - 1) = 1. \quad (7)$$

It doesn't take long to figure out by inspection that $\alpha = 2$ is the solution. And hence the solution for x is

$$x = 5^2 = 25. \quad (8)$$