

Math Diversion Problem 140

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Abstract

Here we use the unipodal algebra to assist in solving the problem, which is given to us on YouTube. Although I'm referring to the series under the name 'olympiad', the problems are from diverse sources as olympiads, entrance exams, SATs, and the like.

I love it when a plan comes together.

— Hannibal Smith, *The A-Team*

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=Klef16FHnk4>

Title: Japanese | Can you solve this?

Presenter: Master T Maths Class

1 The Problem

Given the relation

$$4^x + 25^x = 10^{x+1}, \quad (1)$$

find the values of x over the real numbers.

2 The Solution

The first thing I did was to simplify a bit:

$$4^x + 25^x = 10 \cdot 10^x. \quad (2)$$

Then, if we divide through by 4^x , we can replace a term with variable by a constant.

$$1 + \left(\frac{25}{4}\right)^x = 10 \cdot \left(\frac{10}{4}\right)^x, \quad (3)$$

which simplifies to

$$1 + \left(\frac{5}{2}\right)^{2x} = 10 \cdot \left(\frac{5}{2}\right)^x. \quad (4)$$

Next, we introduce the variable y , as

$$y \equiv \left(\frac{5}{2}\right)^x, \quad (5)$$

by which we can write (4) as

$$y^2 - 10y + 1 = 0, \quad (6)$$

which has roots

$$y = 5 \pm 2\sqrt{6}. \quad (7)$$

Finally, this gives us for x :

$$x_{(+,-)} = \frac{\log(5 \pm 2\sqrt{6})}{\log(5/2)}. \quad (8)$$