Math Diversion Problem 146

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Heuristic note: When it comes time to guess, start by guessing simple.
— The Author

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=CeLKQg9z7LM
Title: Nice Algebra Math Problem
 Presenter: Master T Maths Class
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1 The Problem

Given the relation

$$x^{\sqrt{x}} = 4^2\,, \tag{1}$$

find the values of x over the real numbers.

2 The Solution

In keeping with my admonition above, one could just guess the answer, but I prefer demonstrating the techniques for when guessing isn't enough.

So, I'll make my usual variable transformation in this situation.

$$x = 4^\alpha\,. \tag{2}$$

Then

$$4^{\alpha 4^{\alpha/2}} = 4^2\,, \tag{3}$$

On equating exponents, we have (with some manipulation) that

$$\alpha 4^{\alpha/2} = \alpha 2^\alpha = 2\,, \tag{4}$$

with obvious solution $\alpha = 1$, and thus

$$x = 4\,. \tag{5}$$