Math Diversion Problem 148

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> It is clear that the chief end of mathematical study
> must be to make the students think.
> — John Wesley Young

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=X5iYlxCwGRo&list
     =PLMvuVeOn1Hd_KIT-dsvIVluQQN3pJrlmX&index=46
Title: A Nice Exponential Equation
 Presenter: Master T Maths Class
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1 The Problem

Given the relation

$$x^{x^{1+x}} = 256\,, \tag{1}$$

find the values of x over the real numbers.

2 The Solution

Since the numbers are pretty small, one might save time by guessing. But let's be systematic. First, let's rewrite the given equation as

$$x^{x^{1+x}} = 2^8\,, \tag{2}$$

So, I'll make my usual variable transformation in this situation.

$$x = 2^\alpha\,. \tag{3}$$

Then

$$(2^\alpha)^{(2^\alpha)^{(1+2^\alpha)}} = 2^8\,, \tag{4}$$

which condensces down to

$$2^{\alpha 2^{\alpha(1+2^\alpha)}} = 2^8\,. \tag{5}$$

Equating exponents,

$$\alpha 2^{\alpha(1+2^{\alpha})} = 8 = 2^3\,, \tag{6}$$

with possible integer solution $\alpha = 1$. Therefore,

$$x = 2\,. \tag{7}$$