

Math Diversion Problem 149

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Success is failing 19 times and succeeding the 20th.

— Julia Andrews

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=91jcSSCP8oE>

Title: A Nice Algebra Problem | Math Olympiad

Presenter: SALogic

1 The Problem

Given the relation

$$2^x - 9^x = \sqrt{18^x - 81^x}, \quad (1)$$

find the values of x over the real numbers.

2 The Solution

Let's begin by simplifying the square root.

$$\begin{aligned} 2^x - 9^x &= \sqrt{18^x - 81^x} = \sqrt{9^x \cdot 2^x - 9^x \cdot 9^x} \\ &= \sqrt{9^x \cdot (2^x - 9^x)} = 3^x \cdot \sqrt{2^x - 9^x}. \end{aligned} \quad (2)$$

This can be further simplified to

$$\sqrt{2^x - 9^x} = 3^x, \quad (3)$$

and then one more time

$$2^x - 9^x = 9^x. \quad (4)$$

This can be rearranged to

$$\left(\frac{2}{9}\right)^x = 2. \quad (5)$$

Therefore,

$$x = \frac{\log 2}{\log 2 - 2 \log 3}. \quad (6)$$