Math Diversion Problem 163

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> The definition of a good mathematical problem is the mathematics
> it generates rather than the problem itself.
> — Andrew Wiles

The YouTube video is found at:

1   The Problem

Given the relation
$$2^x - 9^x = \sqrt{18^x - 81^x}\,, \tag{1}$$
find the values of x over the real numbers.

2   The Solution

The first thing to note is that $x = 0$ is a solution.

The given relation can be factored as thus
$$2^x - 9^x = \sqrt{9^x(2^x - 9^x)} = \sqrt{9^x}\sqrt{2^x - 9^x}\,, \tag{2}$$

On squaring both sides and simplifying, we get
$$2^{x-1} = 9^x\,. \tag{3}$$

Since we are not going to be able to solve this with rational values of x, we need to take logarithms.
$$(x - 1)\log 2 = x \log 9\,. \tag{4}$$

On solving for x, we have that
$$x = \frac{\log 2}{\log(2/9)} = -\frac{\log 2}{\log(9/2)}\,. \tag{5}$$

(WolframAlpha disagrees we any solution other than zero.)