Math Diversion Problem 165

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> The human mind has never invented a labor-saving
> machine equal to algebra.
> — J. Willard Gibbs

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=fSfl-Ch1ujI
Title: Japanese | Can you solve this?
 Presenter: Master T Maths Class
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1 The Problem

Given the relation

$$\log_2 x = \log_x 4\,, \tag{1}$$

find the values of x over the real numbers.

2 The Solution

Lemma: A Rule of Logarithms:

$$\log_a b = \frac{\log b}{\log a}\,. \tag{2}$$

Using the logarithm rule above, (1) becomes

$$\frac{\log x}{\log 2} = \frac{\log 4}{\log x} = \frac{2\log 2}{\log x}\,, \tag{3}$$

which then gives us

$$(\log x)^2 = 2(\log 2)^2\,, \tag{4}$$

$$\log x = \pm\sqrt{2}\log 2\,, \tag{5}$$

and thus

$$x = 2^{\pm\sqrt{2}}\,. \tag{6}$$