# Math Diversion Problem 169

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> People often overlook the obvious.
> — Doctor Who

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=MAvbLgfzcww
Title: Nice Exponent Math Simplification
 Presenter: Master T Maths Class
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## 1  The Problem

Given the relation

$$x^x = 3^{2x+27}, \tag{1}$$

find the values of $x$ over the real numbers.

## 2  The Solution

I'll start with my usual change of variable:

$$x = 3^{\alpha}. \tag{2}$$

On substituting this into (1), we have that

$$(3^{\alpha})^{3^{\alpha}} = 3^{2(3^{\alpha})+27}, \tag{3}$$

which becomes

$$3^{\alpha \cdot 3^{\alpha}} = 3^{2(3^{\alpha})+3^3}. \tag{4}$$

On equating exponents, we get

$$\alpha \cdot 3^{\alpha} = 2(3^{\alpha}) + 3^3. \tag{5}$$

After trying various small $\alpha$, I got $\alpha = 3$ to work. Therefore,

$$x = 27. \tag{6}$$