

Math Diversion Problem 170

P. Reany

November 21, 2024

Keep an open mind. That's the secret.
— Doctor Who

The YouTube video is found at:

Source: ?
Title: ?
Presenter: ?

1 The Problem

Given the relation

$$x^7 + \frac{1}{x^7} = 6, \quad (1)$$

what is

$$\phi = x^{21} + \frac{1}{x^{21}} ? \quad (2)$$

2 The Solution

$$\left(x^7 + \frac{1}{x^7}\right)^2 = 36, \quad (3)$$

which becomes

$$x^{14} + \frac{1}{x^{14}} = 34. \quad (4)$$

Then

$$\left(x^7 + \frac{1}{x^7}\right) \left(x^{14} + \frac{1}{x^{14}}\right) = 34 \times 6 = 204. \quad (5)$$

After simplifying this, we get

$$\phi = x^{21} + \frac{1}{x^{21}} = 198. \quad (6)$$