

Math Diversion Problem 214

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Keep an open mind. That's the secret.
— Doctor Who

The YouTube video is found at:

Source: ?
Title: ?
Presenter: ?

1 The Problem

Given the relation

$$x^{2x^6} = 3, \tag{1}$$

find the values of x over the real numbers.

2 The Solution

I'll start with my usual change of variable (in this case, to base 3):

$$x = 3^\alpha. \tag{2}$$

On substituting this into (1), we have that

$$(3^\alpha)^{2 \cdot 3^{6\alpha}} = 3^1, \tag{3}$$

which becomes

$$3^{2\alpha 3^{6\alpha}} = 3^1. \tag{4}$$

On equating exponents, we get

$$2\alpha 3^{6\alpha} = 1. \tag{5}$$

After trying various 'small' integer α 's, I tried reasonable rational numbers, and got $\alpha = 1/6$ to work. Therefore,

$$x = 3^{1/6}. \tag{6}$$