Math Diversion Problem 215

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> You don't understand anything until you learn
> it more than one way.
> — Marvin Minsky

The YouTube video is found at:

1 The Problem

Given the relation

$$9^{4^m} = 4^{9^m} \, , \tag{1}$$

find the values of m over the real numbers.

2 The Solution

I'll start with my usual change of variable (in this case, to base 4). So, let

$$9 = 4^{\alpha} \, . \tag{2}$$

On substituting this into (1), we have that

$$(4^{\alpha})^{4^m} = 4^{9^m} \, , \tag{3}$$

which becomes

$$4^{\alpha 4^m} = 4^{9^m} \, . \tag{4}$$

On equating exponents, we get

$$\alpha 4^m = 9^m \, . \tag{5}$$

Then,

$$\alpha = \left(\frac{9}{4} \right)^m \, . \tag{6}$$

On taking the logarithm of (2) and combining that with this last equation, we have that

$$\frac{\log 9}{\log 4} = \left(\frac{9}{4}\right)^m.$$

(7)

Finally,

$$m = \frac{\log(\log 9 / \log 4)}{\log 9 - \log 4} = \frac{\log(\log 9) - \log(\log 4)}{\log 9 - \log 4}.$$

(8)