Math Diversion Problem 234

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The secret to perseverence is to just keep doing it. - The Author

The YouTube video is found at:

Source: https://www.youtube.com/watch?v=dGLjIPkV9GY Title: An Interesting Nonstandard Equation Presenter: Syber Math

1 The Problem

Given the relation

$$256^x = \frac{1}{x},\tag{1}$$

find the values of x over the real numbers.

I have already solved this problem earlier, but this time I have a new approach to it.

2 The Solution

First, I want to recast (1) to the form

$$x256^x = 1$$
, (2)

and note that

$$256 = 2^8 . (3)$$

On making this substitution, we get

$$x2^{8x} = 1. (4)$$

Thus x and 2^{8x} are inverses of each other. Let's solve this with a table of values.

$$\begin{array}{c|cc} x & 2^{8x} \\ \hline 1/2 & 2^4 = 16 \\ 1/4 & 2^2 = 4 \checkmark \end{array}$$

Table 1: Heuristic: We try various x's as powers of 2.

Therefore

$$x = 1/4. (5)$$