

# Math Diversion Problem 237

P. Reany

December 15, 2024

It is clear that the chief end of mathematical study  
must be to make the students think.

— John Wesley Young  
(So think!)

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=bqFf5R2oofE>

Title: How to solve this nice math Exponential algebra problem

Presenter: Mathematics & Statistics guru

## 1 The Problem

Given the relation

$$a^3 + b^3 + 3ab = 1, \quad (1)$$

find the values of  $a + b$  over the real numbers. For convenience, let

$$\phi \equiv a + b. \quad (2)$$

## 2 The Solution

Let

$$\phi^3 - 0 = (a + b)^3 - (a^3 + b^3 + 3ab - 1) \quad (3a)$$

$$= a^3 + 3a^2b + 3ab^2 + b^3 - (a^3 + b^3 + 3ab - 1) \quad (3b)$$

$$= 3a^2b + 3ab^2 - 3ab + 1 \quad (3c)$$

$$= 3ab(b + a - 1) + 1 \quad (3d)$$

$$= 3ab(\phi - 1) + 1 \quad (3e)$$

By inspection, we can see that  $\phi = 1$  is a solution.