

# Math Diversion Problem 242

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I love it when a plan comes together.  
— Hannibal Smith, *The A-Team*

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=EMu-kYY5rdE>  
Title: Is  $e^x = \ln(x)$  solvable?  
Presenter: blackpenredpen

## 1 The Problem

Given the relation

$$e^x = \ln x, \tag{1}$$

find the values of  $x$ .

## 2 The Solution

I intend to use the Lambert  $W$  function lemma, that, if

$$z \ln z = B, \tag{2}$$

then

$$\ln z = W(B). \tag{3}$$

OK, I'll start by multiplying through by  $x$ :

$$xe^x = x \ln x. \tag{4}$$

Now, a curious thing happens when we take the Lambert  $W$  function across this equation; we get

$$x = \ln x. \tag{5}$$

Next, we do some algebra until we reach this form:

$$-1 = x^{-1} \ln x^{-1}. \tag{6}$$

Applying the Lambert  $W$  function once more, we have that

$$W(-1) = \ln x^{-1}, \quad (7)$$

which gives

$$x^{-1} = e^{W(-1)}, \quad (8)$$

and then

$$x = e^{-W(-1)}. \quad (9)$$

And with an identity, we can also write this as

$$x = \frac{W(-1)}{-1} = -W(-1). \quad (10)$$