Math Diversion Problem 243

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First things first...But not necessarily in that order.
— Doctor Who

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=nWWocG37KzI
Title: Oxford University Pure Mathematics Admission Exam
 Presenter: Super Academy
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1 The Problem

Given the relation

$$\sqrt{x}^{x} = 2^{x+16}\,, \tag{1}$$

find the values of x.

2 The Solution

I'll start by squaring the Given equation:

$$x^{x} = 2^{2x+32}\,. \tag{2}$$

Next, I'll make the change in variable:

$$x = 2^{\alpha}\,, \tag{3}$$

which brings (2) to the form

$$(2^{\alpha})^{(2^{\alpha})} = 2^{2(2^{\alpha})+32}\,. \tag{4}$$

On equating the exponents, we get

$$\alpha 2^{\alpha} = 2(2^{\alpha}) + 32\,. \tag{5}$$

Or,

$$(\alpha - 2)2^{\alpha} = 32 = 2^5 \,. \tag{6}$$

One more time:

$$(\alpha - 2)2^{\alpha - 5} = 1 \,. \tag{7}$$

A little trial and error gives us $\alpha = 4$, therefore,

$$x = 16 \,. \tag{8}$$