Math Diversion Problem 247

P. Reany

December 21, 2024

People often overlook the obvious.
— Doctor Who

The YouTube video is found at:

```
Source: https://www.youtube.com/watch?v=8n1J4goP6vM
Title: A Nice Equation | Problem 433
 Presenter: aplusbi
```

1 The Problem

Given the relation

$$\frac{\ln z}{z} = \frac{\pi}{2}\,, \tag{1}$$

find the values of z.

2 The Preparation

I intend to use the Lambert W function lemma, that, if

$$y \ln y = B\,, \tag{2}$$

then

$$\ln y = W(B)\,. \tag{3}$$

3 The Solution

Before we can apply the lemma, we must place the Given relation into a proper form. First,

$$z^{-1} \ln z = \frac{\pi}{2}\,, \tag{4}$$

then

$$-z^{-1}\ln z = z^{-1}\ln z^{-1} = -\frac{\pi}{2}\,. \tag{5}$$

Now we apply the lemma:

$$\ln z^{-1} = W(-\frac{\pi}{2})\,, \tag{6}$$

And the result is

$$z = e^{-W(-\pi/2)}\,. \tag{7}$$

But what is $W(-\pi/2)$? What is the value of xe^x that gives us $-\pi/2$?

$$xe^x = -\pi/2\,. \tag{8}$$

How about $x = i\pi/2$?

$$(i\pi/2)e^{i\pi/2} = (i\pi/2)(+i) = -\pi/2\,. \tag{9}$$

However, $x = -i\pi/2$ also works.

$$(-i\pi/2)e^{-i\pi/2} = (-i\pi/2)(-i) = -\pi/2\,. \tag{10}$$

Thus,

$$W(-\pi/2) = \pm i\pi/2\,. \tag{11}$$

And finally,

$$z = e^{\pm i\pi/2} = \pm i\,. \tag{12}$$