Math Diversions, Problem 32

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People often overlook the obvious. — Doctor Who

1 Problem

The YouTube video is found at:

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https://www.youtube.com/watch?v=LGOAgRNN1Po
  Titled: Cambridge University Exponential Problem ||
      Admission Interview tricks
  Presenter: Super Academy
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Given the relation

$$x^x = 2^{8+2x} \,, \tag{1}$$

find the value of x.

2 Solution

Look at the RHS of (1): what do you see? I see the product of a whole bunch of 2's. So, what's true of prime factors on the RHS must also be true of the LHS. Therefore x^x must be the product of a bunch of 2's. The most obvious way to deal with this is to write

$$x = 2^y \,. \tag{2}$$

Thus, (1) becomes

$$(2^y)^{2^y} = 2^{8+2 \cdot 2^y} \,, \tag{3}$$

But we can rewrite the RHS to be $(2^y)^{\frac{8+2 \cdot 2^y}{y}}$. Therefore, (3) becomes

$$(2^y)^{2^y} = (2^y)^{\frac{8+2 \cdot 2^y}{y}} \,. \tag{4}$$

On comparing exponents, we get

$$2^y = \frac{8 + 2 \cdot 2^y}{y} \,. \tag{5}$$

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But this can be rewritten as

$$(y-2)2^y = 8. \tag{6}$$

But these are all small numbers, so let's just try some small numbers. For $y = 1$ we get a negative number on the LHS. For $y = 2$ we get zero on the LHS. For $y = 3$ we get a solution. Higher values of y won't give us another solution.

Now, since $y = 3$, then

$$x = 2^3 = 8. \tag{7}$$