

Math Diversions, Problem 34

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September 8, 2024

People often overlook the obvious. — Doctor Who

1 Problem

The YouTube video is found at:

<https://www.youtube.com/watch?v=a4VP42kz7d0>
Titled: Working with Logarithmic Expressions
Presenter: SyberMath

Given the relations

$$6 \log a = 4 \log b = 3 \log c. \quad (1)$$

find the numeric value of

$$\log_{ab} c. \quad (2)$$

2 Solution

From (1) we can write

$$\log a = \frac{1}{2} \log c, \quad (3a)$$

$$\log b = \frac{3}{4} \log c. \quad (3b)$$

Next, let's set

$$x \equiv \log_{ab} c, \quad (4)$$

and use this equation as the powers of the number ab :

$$(ab)^x = (ab)^{\log_{ab} c} = c. \quad (5)$$

Now we take the logarithm across this equation.

$$\log c = x(\log(ab)) = x(\log a + \log b). \quad (6)$$

Hence,

$$x = \frac{\log c}{\log a + \log b} = \frac{\log c}{\frac{1}{2} \log c + \frac{3}{4} \log c} = \frac{1}{\frac{1}{2} + \frac{3}{4}} = \frac{4}{5}. \quad (7)$$