

Math Diversions, Problem 35

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People often overlook the obvious. — Doctor Who

1 Problem

The YouTube video is found at:

<https://www.youtube.com/watch?v=C5HTp1Gs1AQ>
Titled: Evaluating A Nice Polynomial | Math Olympiads
Presenter: SyberMath

Given the relation

$$b^3 - b = 1. \tag{1}$$

find

$$b^5 - b^4. \tag{2}$$

2 Solution

From (1) we have that

$$b^3 = b + 1. \tag{3}$$

Then,

$$\begin{aligned} b^5 - b^4 &= b^2(b^3) - b(b^3) \\ &= b^2(b + 1) - b(b + 1) \\ &= b^3 + b^2 - b^2 - b \\ &= b^3 - b \\ &= 1. \end{aligned} \tag{4}$$