

# Math Diversions, Problem 36

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People often overlook the obvious. — Doctor Who

## 1 Problem

The YouTube video is found at:

<https://www.youtube.com/watch?v=LtuCtaW3CYw>  
Titled: A very tricky math question with factorial  
Presenter: Higher Mathematics

Given the relation

$$6! \times 7! = x!, \tag{1}$$

find  $x$ .

## 2 Solution

Let's think about this. On the LHS of (1) there is a collection of different primes. The RHS must therefore contain the exact same primes to the same orders. In particular, the largest prime on the LHS is 7. This means that the largest prime on the RHS must also be 7.

So, considering  $x$ 's according to this constraint,  $x = 8$ ,  $x = 9$ , and  $x = 10$  are possible solutions, but  $x = 11$  is not, because 11 would be a prime on the RHS that is not on the LHS. So, let's start with  $x = 10$  and go backwards.

$$10! = 10 \cdot 9 \cdot 8 \cdot 7!. \tag{2}$$

This looks promising. If  $10!/7! = 6!$ , we're finished.

$$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5 \cdot 3^2 \cdot 2^4 \cdot 1. \tag{3}$$

But

$$10!/7! = 10 \cdot 9 \cdot 8 = 5 \cdot 3^2 \cdot 2^4 \cdot 1. \tag{4}$$

Since these last two values are equal,  $x = 10$  is correct. (By the way,  $x = 10$  is the smallest value that  $x!$  will give us a factor of 5.)