Math Diversions, Problem 39

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People often overlook the obvious.
— Doctor Who

1 Problem

The YouTube video is found at:

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https://www.youtube.com/watch?v=GX7MzC0_2oM
 Titled: Harvard University Aptitude Test Tricks
       || Algebra Problem
 Presenter: Super Academy
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Given the relations

$$5^x + 5^y = 15750\,, \tag{1a}$$
$$x + y = 9\,, \tag{1b}$$

find the value of x and y.

2 Solution

Given all the number of 5's in (1a), it seemed reasonable to me to see how many factors or 5 are in 15,750. I guessed there were three and that was correct.

$$15,750 = 5^3 \cdot 126\,. \tag{2}$$

Thus, (1a) becomes

$$5^{x-3} + 5^{y-3} = 126\,. \tag{3}$$

Given that we are working with integers, at this point in the analysis, we are supposed to analyze how we can get the sum of powers of 5's to equal an even number that does not end in 0, which wouldn't help anyways. Odd + odd is even, but in this case, one of those odds must be unity.

Without loss of generality, let that be because we choose $x - 3 = 0$. Since $x = 3$, then from (1b), $y = 9 - 3 = 6$. The only thing left to do is to try these values to see if they actually work:

$$5^3 + 5^6 = 125 + 15,625 = 15,750\,. \tag{4}$$