Math Diversions, Problem 54

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People often overlook the obvious.
— Doctor Who

1 Problem

The YouTube video is found at:

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 Titled: ????
 Presenter: ????
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Given the relations

$$\log y = \log_x(2x - 5)\,, \tag{1a}$$

$$\log x = \log_y(x + 15)\,, \tag{1b}$$

find the values of x, y.

2 Solution

The Big Rule of Logarithms when changing base is:

$$\log_a b = \frac{\log_c b}{\log_c a}\,, \tag{2}$$

wher c is an arbitrary positive real number.

We can modify our original equations to:

$$\log y = \log_x(2x - 5) = \frac{\log(2x - 5)}{\log x}\,, \tag{3a}$$

$$\log x = \log_y(x + 15) = \frac{\log(x + 15)}{\log y}\,. \tag{3b}$$

Thus (3a) and (3b) become

$$\log x \log y = \log (2x - 5), \tag{4a}$$
$$\log y \log x = \log (x + 15). \tag{4b}$$

From these we conclude that

$$\log (2x - 5) = \log (x + 15). \tag{5}$$

Hence,
$$(2x - 5) = (x + 15), \tag{6}$$

and thus
$$x = 20. \tag{7}$$

Solving for y, we get

$$y = \log^{-1} \frac{\log 35}{\log 20}. \tag{8}$$