Math Diversion Problem 59

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September 22, 2024

I love it when a plan comes together.
— Hannibal Smith, *The A-Team*

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=Q6vFA8FqKzA
 Title: Indian 1 Olympiad Math Algebric Exponential
   | Find x=?
  Presenter: Math Master TV
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1 The Problem

Given the relation

$$6^x + 9^x = 2^{2x+1}\,, \tag{1}$$

find the integer values of x.

2 The Solution

First, let's rearrange (1) to get

$$9^x = 2^{2x+1} - 6^x\,, \tag{2}$$

which can be rewritten as

$$9^x = 2^{2x+1} - 2^x 3^x = 2^x(2^{x+1} - 3^x)\,. \tag{3}$$

Now, for all non-negative integers, the LHS will be odd. But for all positive integers, the RHS will be even. Thus, we have only one choice for x:

$$x = 0\,. \tag{4}$$