

Math Diversion Problem 68

P. Reany

September 29, 2024

The only way to learn mathematics is
to do mathematics.
— Paul Halmos

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=W0dJwFFGXRw>

Title: maths olympiad question || An algebraic
exponential problem

Presenter: Maths Curiosity

1 The Problem

Given the relation

$$2^a + 2^b + 2^c = 148, \quad (1)$$

find the integer values of a, b, c .

2 The Solution

Since a, b, c enter the problem symmetrically, we need to set a constraint to solve this problem. Without loss of generality, we assume that

$$a \leq b \leq c. \quad (2)$$

Okay, we begin by rewriting the given equation (1) to get

$$2^a + 2^b + 2^c = 2^2 \cdot 37. \quad (3)$$

On dividing through by 2^2 , we have that

$$2^{a-2} + 2^{b-2} + 2^{c-2} = 37. \quad (4)$$

Now, since the RHS is an odd number, we have to force the LHS to be odd. To accomplish this, we set $a - 2 = 0$, which leaves us with

$$1 + 2^{b-2} + 2^{c-2} = 37, \quad (5)$$

which becomes

$$2^{b-2} + 2^{c-2} = 36 = 2^2 \cdot 3^2. \quad (6)$$

So, now we divide through by 2^2 :

$$2^{b-4} + 2^{c-4} = 3^2 = 9. \quad (7)$$

We need to set $b - 4 = 0$, to get

$$1 + 2^{c-4} = 3^2 = 9, \quad (8)$$

from which we get

$$2^{c-4} = 8. \quad (9)$$

Finally, we need to set $c - 4 = 3$. This leaves us with the values:

$$a = 2 \quad b = 4, \quad c = 7. \quad (10)$$