Math Diversion Problem 84

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> Once you learn patience, your options suddenly expand.
> — Robert Greene

The YouTube video is found at:

1   The Problem

Given the relation

$$a^4 = (a-1)^4 \,, \tag{1}$$

find the values of a.

2   The Solution

There are two obvious immediate implications one can draw from (1). The first is that $\frac{1}{2}$ is a solution! The second is that the equation has only three solutions because, once the RHS of (1) is expanded, we'll find that the terms a^4 on each side will cancel out, leaving just a cubic behind.

So, let's do expand the RHS:[1]

$$a^4 = a^4 - 4a^3 + 6a^2 - 4a + 1 \,, \tag{2}$$

and this becomes

$$4a^3 - 6a^2 + 4a - 1 = 0 \,. \tag{3}$$

Now, we already observed that $1/2$ is a root of the given equation, and that has to apply to (3) as well. So, after doing a long division on $4a^3 - 6a^2 + 4a - 1$, I got

$$(a - \tfrac{1}{2})(4a^2 - 4a + 2) = 0 \,. \tag{4}$$

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[1] I'll use Pascal's Triangle for the coefficients.

And this leaves us with

$$4a^2 - 4a + 2 = 0\,,\tag{5}$$

whose solutions are

$$a = \frac{1 \pm i}{2}\,,\tag{6}$$

and those are the last two of the three solutions.