Math Diversion Problem 85

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> Keep an open mind. That's the secret.
> — Doctor Who

The YouTube video is found at:

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Source: ?
Title: ?
 Presenter: ?
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1 The Problem

Given the relations

$$\sqrt{x} + y = 7\,, \tag{1a}$$
$$x + \sqrt{y} = 11\,, \tag{1b}$$

find the values of $x, y \in \mathbb{R}$.

2 The Solution

I chose to begin with a variable substitution into (1a,1b).

$$a = \sqrt{x}\,, \tag{2a}$$
$$b = \sqrt{y}\,. \tag{2b}$$

Then, the given equations become

$$a + b^2 = 7\,, \tag{3a}$$
$$a^2 + b = 11\,, \tag{3b}$$

On solving (3b) for b and putting that into (3a), we get

$$a + (11 - a^2)^2 = 7\,, \tag{4}$$

with real solution

$$a = 3\,. \tag{5}$$

This gives us $x = 9$. We then get for b

$$b = 2\,. \tag{6}$$

And this leaves us with

$$4a^2 - 4a + 2 = 0\,, \tag{7}$$

and this gives us

$$y = 4\,. \tag{8}$$