

# Math Diversion Problem 91

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In mathematics, the art of proposing a question must be  
held of higher value than solving it.  
— Georg Cantor

The YouTube video is found at:

Source: <https://www.youtube.com/watch?v=mh5UPPaJLXY>  
Title: Can You Solve This 12-Year-Old's Math  
Problem from China?  
Presenter: Global Maths

## 1 The Problem

Given the relation

$$(x + 100)^2 = 2x + 199, \quad (1)$$

find the values of  $x$ .

## 2 The Solution

The given is a quadratic in  $x$ , so it has two roots in the complex numbers (though a root can be a double root). I decided to try this problem because I had this idea that with a suitable variable substitution, I could make the final quadratic equation simple to solve. Let's see.

Let's replace the  $x$  in (1) with  $y - a$ :

$$(y - a + 100)^2 = 2(y - a) + 199. \quad (2)$$

On expanding this we get

$$y^2 - 2ay + a^2 + 200y - 200a + 10,000 = 2y - 2a + 199. \quad (3)$$

Now, here's the point of the variable substitution: I will choose  $a$  to force the linear term of this last equation to go to zero. Setting the components of the linear terms from (3) to make them vanish, we get

$$-2a + 200 - 2 = 0, \quad (4)$$

So, if I set  $a = 99$ , that will force all the linear terms to drop out (effectively), leaving us with

$$y^2 + a^2 - 200a + 10,000 = -2a + 199. \quad (5)$$

But this equation has the solution

$$y = 0. \quad (6)$$

So, what does that imply for  $x$ , where  $x = y - a$ ?

$$x = -99. \quad (7)$$