Math Diversion Problem 95

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With me, everything turns into mathematics.
— Rene Descartes
(P.S. I calculate; therefore I am.)

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=Xw8VBHXK81M
Title: Nice Exponential Math Problem |
          Harvard Entrance Exam Question
 Presenter: SchoolClass Math
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1 The Problem

Given the relation

$$20^k \cdot 50^k = 8\,, \tag{1}$$

find the values of k.

2 The Solution

There are many ways to continue from here. I chose this way. The given relation becomes

$$(20 \cdot 50)^k = (1000)^k = 8\,. \tag{2}$$

Take the logarithm base 10 of both sides.

$$k \log 1000 = k \log 10^3 = 3k = \log 8\,. \tag{3}$$

From this we get

$$k = \log 8^{1/3} = \log 2\,. \tag{4}$$