

Word Problem Write-Up: Using Scheme

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Abstract

In this write up on algebra word problems, I will present my own system that I developed for over four decades. I call this system *Scheme*, and it has similar presentations by other people.

I'm a visual thinker, really bad at algebra. There's others
that are a pattern thinker. These are the
the music and math minds.
— Temple Grandin

1 Preface

I want to begin by addressing the quote above by Temple Grandin. Her characterization is probably shared by most people, which is why I quoted it here. I suggest that her perception of the discipline of algebra being void of visual representation is a misconception of what can be done in the effort. And it was a misconception I shared as a teenage problem solver, myself.

However, the truth is that you can put in visual aids to help you solve your algebra word problems as much as you like. This write up and the many word problems it addresses will provide a lot of examples on how this can be done. In Scheme, a particular way of expressing visual aids is promoted.

So, why don't algebra textbooks use more visual aids? Well, I'm not up-to-date on modern algebra books, but it stands to reason that a big reason visual aids are not more prevalent in textbooks is because they take up a lot of room and therefore add to the cost of the book. Also, a visual aid in a textbook has to look professionally done and that's expensive. On the other hand, the visual aids in my papers aren't professionally done and it shows. But they are effective and my math papers are free.

2 Introduction

In Scheme, we don't just start at the beginning, we start one step prior to the beginning.¹

Zeroth Rule of Problem Solving

Make any assumption necessary in order to solve the problem in a reasonable amount of time with a reasonable amount of effort.

Back when I was taking a class in programming, the instructor advised us to always make our assumptions explicit, not only to prevent problems, but also to find them if they occur unexpectedly. If you find yourself frustrated while troubleshooting your faulty solution to a problem, you may be stymied because the source of the trouble may lie in one of those assumptions you didn't bother make explicit in the formal presentation of the solution.²

The top-down approach

Our next heuristic is also inspired by my formal education in computer programming: The top-down approach. We think of solving a word problem as first translating the word problem from English (or some natural language) into algebraic form and then solving the system of equations (or inequalities) thus derived. The *art* of solving word problems is not in the algebra per se, because that is just a matter of learning techniques that anyone can memorize and master with some practice. The art comes from converting the English words (or some natural language) into the algebra, by a stepwise process, the highest level of which could be likened to *pseudocode* in computer programming — the philosophy of which is to suppress details as long as possible.

This is the technique I like to use (in brief): Find the totals and parts and/or the invariants, etc, and then employ them in English sentences that are complete but don't immediately appear as 'algebraic'. Then refine the sentences step-by-step until the end result is an algebraic equation or inequality. Then solve the system.

3 Word Problem

A jar containing nickels and dimes has \$1.05 worth of coins in it. If the jar contains exactly 16 coins, how many are nickels and how many are dimes?

Now, as it stands, this problem is ambiguous. What we need to know is if there are *only* nickels and dimes in the jar. By Zeroth Rule (above) then it is reasonable to assume that there are only nickels and dimes in the jar.

¹Much of the presentation here is copied from my many prior papers on solving word problems using Scheme.

²In logic, one uses the term *enthymeme* to refer to an argument with a tacit assumption.

Solution Part 1: Conceptualizing the problem

Most algebra word problems will contain at least one total, so, generally speaking, unless there is an obvious reason *not* to start our solution by looking for a total, let's do so.³

There are two obvious totals in the given information. The first is the total money in coins, being \$1.05. The second is the total number of coins in the jar. Now, this is where the assumption that there are only nickels and dimes in the jar comes in handy. You see, our procedure is to find a total, discover all of its parts, add those parts together, and set that sum equal to the total. Since we assume that the 'coin parts' exist only as nickels and dimes, we begin our formulation of this equation in the simple, easy-to-understand 'pseudocode' form of

$$(\text{dollar value in nickels}) + (\text{dollar value in dimes}) = \$1.05. \quad (1)$$

As part of our conceptualization of this process of sorting the coins by type, we can abstract this invariant process in the figure below.

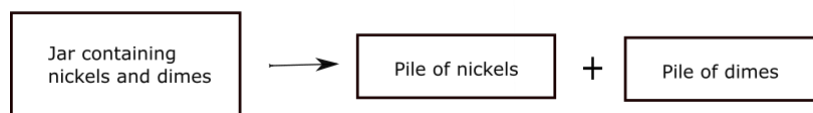


Figure 1. This graphic represents our imagined sorting of all the coins into a pile of nickels and a pile of dimes, leaving invariant the number of each. Our first real visual aid.

Now, before we make our first step-wise refinement of Eq. (1), let's ask a more general question in preparation. What does it mean to calculate the dollar value of a pile of coins of a single type? It means to count the number of coins and then multiply this number by the dollar value of each coin:

$$(\text{value of a single coin})(\#\text{coins}) = \text{value of pile of coins}, \quad (2)$$

where we have placed the conversion factor to the left of the number of coins, which is customary but not necessary

Time out, please!

A *conversion factor* is a rate of change; specifically, the rate of change of things in one unit (in the numerator) into things into some other unit (in the denominator), and vice versa. Perhaps you think that it's more proper to restrict our notion of a conversion factor to converting between things of 'like nature',

³When searching for 'parts', we need to find enough of them to add up to the total we seek. However, we need to be sure that the parts are mutually exclusive (they don't intersect) so that we don't exceed the total. This is what is meant by the expression 'mutually exclusive and collectively exhaustive'.

such as in the case of converting between inches to feet or yards to meters — all dealing with lengths or distances in this particular example. But this is an unhelpful and unnecessary restriction. What we really want is to form a conceptual basis for solving algebra problems in which the least number of primitive notions conceivable can cover the most number of particular cases.

Let's consider two other examples. First, velocity. Velocity is indeed a *rate of change*, but it is also a conversion factor, changing timelike things into distancelike (or spacelike) things.

Second, one of my favorite examples of both 'totals being the sum of their parts' and the use of conversion factors rolled into the familiar example of what we owe on our grocery purchases: the total cost of groceries. To simplify matters, we'll assume we're buying at the grocery store two types of untaxed groceries and paying with cash. Suppose we are buying four of one kind of apple at \$0.50/apple and three cans of peas at \$1.14/can. Again, we begin with a 'total' equation:

$$\text{total grocery bill in \$} = (\$ \text{ cost of apples}) + (\$ \text{ cost of cans of peas}). \quad (3a)$$

A stepwise refinement of this last equation, gives

$$\begin{aligned} \text{total \$ grocery bill} &= \frac{\$0.50}{\text{apple}}(4 \text{ apples}) + \frac{\$1.14}{\text{can of peas}}(3 \text{ cans}) \\ &= \$2.00 + \$3.42, \end{aligned} \quad (3b)$$

where each of these last two terms is called a *subtotal*. And the total cost of our groceries is \$5.42.

At the conceptual level, what is going on here? The conversion factors are telling us how much (many) goods we can take from the store converted into how much cash we must leave at the register.

Returning to our coin problem, the value of a single unspecified type coin is $\frac{\$X.YZ}{1 \text{ coin}} = \frac{\$X.YZ}{\text{coin}}$, dropping the superfluous 1 in the denominator. Now, just to be a bit exotic, let's say the coin in question is a \$20 gold piece, and we have twenty of them. Then Eq. (2) becomes

$$\left(\frac{\$20.00}{\text{coin}}\right)(20 \text{ coins}) = \$400.00. \quad (4a)$$

In the language of 'units' in algebra, we say that in the above equation the coin unit has 'canceled out'. We could have made this more explicit by writing

$$\left(\frac{\$20.00}{\cancel{\text{coin}}}\right)(20 \cancel{\text{coins}}) = \$400.00. \quad (4b)$$

Time in. (Thanks for your patience!)

Our first **step-wise refinement** on Eq. (1) yields

$$(\$ \text{ value of a nickel})(\#\text{nickels}) + (\$ \text{ value of a dime})(\#\text{dimes}) = \$1.05. \quad (5)$$

We still have not yet introduced any variables in this algebra problem. To me, Scheme represents a complete paradigm shift in regards to the introduction of variables. As a novice word-problem solver in high school, I figured that the way to start to solve a word problem is to determine the right unknowns from the very beginning. but Scheme turns this approach on its head. Now, I look first for the high-level equations to write down. Then I refine them stepwise, until I reach the point when introducing variables seems natural and usually ‘unforced’.

Let’s introduce variables now, setting $D = \#dimes$ and $N = \#nickels$. Then, for our next **step-wise refinement** we get

$$\left(\frac{\$0.05}{\text{nickel}}\right)(N \text{ nickels}) + \left(\frac{\$0.10}{\text{dime}}\right)(D \text{ dimes}) = \$1.05. \quad (6)$$

Thus, we have two unknowns but only one equation. So, we need one more equation in N and D to be able to solve for the two unknowns. Now, what I am about to write may seem terribly pedantic, but if we were writing these equations in a computer language with strong typing requirements, we would have to pay very close attention to the units of our subtotals. We actually did that properly when we considered the units in the subtotals of the ‘value’ equations above. But now it’s time to write down the total coins equation:

$$(\# \text{ nickel coins}) + (\# \text{ dime coins}) = \text{total coins} = 16 \text{ coins}. \quad (7)$$

In other words, coins + coins = coins.

4 Solution Part 2: Solving the system

Fortunately, I don’t intend to be this pedantic about units in future word problems, but I did want to be very clear about the meaning of ‘adding subtotals to get a total’ once. Now, I’ll strip the equations of *all* units and write

$$1.05 = 0.05N + 0.10D, \quad (8a)$$

$$16 = N + D. \quad (8b)$$

This system has the unique solution $N = 11$ and $D = 5$. Thus, there are 11 nickels and 5 dimes in the jar.

Perhaps you’re wondering why I didn’t use x and y instead of N and D . Mathematically speaking, the choice of variable identifiers is arbitrary, so long as they can be distinguished. A more serious issue to deal with is the recommendation of some authors to choose the two variables as x and $16 - x$, what I refer to as ‘accelerated substitution’. Although I can see the advantage of this for some people in some problems, I find it a bad habit, especially for the novice. I’m trying to teach a technique for solving algebra word problems that won’t break down when the number of variables increases beyond two.

5 The Scheme

You have to know what to look for, so you can spot it.

— Papago Indian drug-enforcement
border scout

The Scheme I cobbled together over the last three decades goes something like this:

1. If there are any totals or parts of a total lying around, put them into an equation (or into an inequality, if appropriate). Then $\text{Total} = \sum_i \text{Parts}_i$.
2. Is there some invariant Inv evidently holding from an initial state to the final state of a before-and-after process? If so, write $\text{Inv}_i = \text{Inv}_f$. (That's an equation!)
3. Is there a common or problem-specific formula to use? Such as from physics, chemistry, etc., or from mathematics, like from geometry or from number theory, such as for the summation of a series or for a weighted average of a set of numbers, or the greatest common factor or least common multiple of two or more numbers.

4. Is there a proportion given? A *proportion* is the stated equality of two ratios.

$$\frac{a}{b} = \frac{c}{d}.$$

5. Are there one or more linear or quadratic relations given or detected? If so, write them down.
6. Is there a function given, dependent on one or more discrete variables, each over finite ranges? If so, the problem might be solvable by *exhaustive search*. For examples, the point of the problem may be to find all allowable values of the function satisfying certain given constraints, or it may be a simple matter of finding a minimum or maximum value of the function on its domain. This may not seem like a legitimate algebra problem, but I have found such problem types in algebra problem sets. So, as a practical matter, it's good to keep it in the list of heuristics.
7. And, add as many additional heuristics as you like.

I have a number of comments before ending this section. First, the above set of heuristics did not mention anything about either *units* or *conversion factors*, yet they come up often in word problems and the problem solver must know how to deal with them. Second, most word problems I have solved over the years have used at least one version of a 'total = the sum of its parts'. And many of the interesting problems I've solved have used some invariant in a before-and-after process. Third, I predict that, in a well-crafted word problem, it will be nearly impossible to find an equation of the form of either items 1) or 2) above

that will *not* be useful to finding the solution to the problem. Therefore, unless a particular problem strongly suggests starting elsewhere, begin the problem solving by searching for equations of the forms found in items 1) and 2) above.

So why is Scheme so effective for solving word problems? Because it encapsulates what is true of *every* word problem at the highest level of abstraction! After all, the process of solving a word problem can be reduced to a two-step process: Find a complete set of equations and/or inequalities that characterize the problem and then solve that set simultaneously. Scheme facilitates accomplishing that first step of the process.

6 The Preparation for ‘mixed-rate’ problems

In the domain of algebra word problems, I define a ‘mixed-rate’ problem as a word problem that tells of two or more ‘machines’ that work together — usually simultaneously, but not always — to accomplish a single goal, task, or job. I will usually use the word ‘job’.

So, you say, “What do you mean ‘machines’?! I know lots of such word problems that speak of people doing these so-called ‘jobs’, and people are **not** machines!”

Well, if you think that way, then you’ll have to come up with a separate version of this definition, which I deem to be unnecessary. If you can’t easily generalize situations through abstractions to encompass ever wider realms of the subject, then you are needlessly hindering your advancement in mathematics.

In essence, what’s the difference between two or more people painting a house at different rates, and two or more printing machines printing out a print job at different rates, and two or more fonts at different point sizes filling out a page (at different rates), and two or more water pumps filling or emptying a tank at different rates, and two or more kinds of nuts supplying protein to the final mixture at different rates, and how to invest an initial amount of money in two or more investment options, subject to legal constraints, which produce interest on the investments at different rates? I could list many more examples, I’m sure. The point is that, conceptually speaking, these are **all the same problem**, save for minor technical issues (that usually reveal themselves as mathematical constraints).

So here’s a perfect example of how to use the heuristic that ‘Every total is equal to the sum of its parts’: Starting with the given that there are two different ‘machines’, M1 and M2, working at different rates to complete a single job, what can we always conclude? We can conclude this:⁴

$$1 \text{ job} = (\text{PJDB M1}) + (\text{PJDB M2}), \quad (9)$$

⁴If the cooperating ‘machines’ all work at the same rate, then, by definition, the problem is not a ‘mixed-rate’ problem. But if one knows how to solve ‘mixed-rate’ problems, one should be able to solve single-rate problems.

where PJDB stands for ‘part of job done by’.

Note: The following problems are just a few that I have published some time ago on my website.

7 Word Problem #4.1

Consider the following problem: Printer #1 can print a 100 copies of a document in 3.4 hours and Printer #2 can print out the same print job in 2.5 hours. How long will it take for the print job to complete if both printers work on the job together, starting and stopping at the same time?

Solution Part 4.1.1: Conceptualizing the problem

We introduce the shorthand ‘part of job done by’ \rightarrow PJDB. Then our highest-level equation is

$$1 \text{ job} = (\text{PJDB Printer 1}) + (\text{PJDB Printer 2}). \quad (10)$$

Let R_1 be the average rate at which Printer 1 can work, which is 1 job/3.4 hours. Likewise, R_2 is the average rate at which Printer 2 can work, which is 1 job/2.5 hours. Now, the most general expression we can write for the refinement of the last equation is⁵

$$1 \text{ job} = R_1 T_1 + R_2 T_2, \quad (11)$$

where T_1 and T_2 are the respective times that Printer 1 and Printer 2 are operating. For the current problem, these two times are equal and they are equal to the total time the print job takes, but the last equation is the most general for two printers. So, for our current problem, let’s set this common time equal to T and suppress units, to get

$$1 = R_1 T + R_2 T = (R_1 + R_2) T. \quad (12)$$

Perhaps this equation is beginning to look familiar to you from your previous algebra experience (such as in the problems given in the SAT, GRE, or LSAT). Solving for T , we get

$$T = \frac{1}{R_1 + R_2}. \quad (13)$$

If this equation looks close to what you remember, but not quite right, that could be because you’re used to thinking of the rates in units of [hours/job], the inverse of the units for R . So, letting $G_1 = R_1^{-1}$ and $G_2 = R_2^{-1}$, we get⁶

$$T = \frac{1}{G_1^{-1} + G_2^{-1}} = \frac{G_1 G_2}{G_1 + G_2}. \quad (14)$$

⁵We are employing the Zeroth Rule of Problem Solving to make the simplifying assumption that the average rate will be accurate for arbitrarily long or short time intervals.

⁶By my choosing G for the inverse of R , I have intentionally made a formal analogy between the variables here and resistance and conductance in parallel electric-circuit analysis.

Solution Part 4.1.2: Solving the problem

Anyway, using the given values of R_1 and R_2 plugged into (13), we get $T = 1.44$ hours \approx 1 hour 26 minutes.

The first question we should ask of this answer is if it's reasonable, or as they say, is it 'in the ball park'. For starters, it's less than the shorter individual time, so that's a good sign. Second, what answer would we get if the problem were changed to two printers working at 3 hours each (that value is roughly the average of the two job times)? Both working together should give half the time of either working alone, which is 1.5 hours. And this answer is close to 1.44 hours.

Let's change the problem slightly to make a point. The boss sends me an email telling me that the print job isn't due till next week, and that I can have Printer 1 all day Wednesday and Printer 2 all day Thursday. Is that OK? I email back that's fine. Here's why: I just run Printer 1 for the scheduled job for 1.44 hours on Wednesday and run Printer 2 for its scheduled job for 1.44 hours on Thursday. The print job is finish on Thursday, even though the two printers did not run at the same time.

8 Word Problem #4.2

The main oil pump at an oil refinery can fill a tanker in 2 hours. The engineer in charge is concerned that the main pump is in need of repair and he doesn't want to stress it too much. He has a pumping window of 4 hours to fill a tanker before the main pump goes off-line. He also has available a slower pump that can fill the tanker in 6 hours. If the engineer wants to start the tanker job with the slower pump and then add in the main pump at the last possible moment, and then run both simultaneously until the tanker is full, how long will the main pump be used?

Solution Part 4.2.1: Conceptualizing the problem

Let's refer to the faster pump (the main pump) as Pump 1, and the other pump as Pump 2. We have 1 job to be accomplish, split into two contributions:

$$1 \text{ [job]} = (\text{PJDB Pump 1}) + (\text{PJDB Pump 2}). \quad (15)$$

And the next refinement gives us (suppressing units)

$$1 = R_1 T_1 + R_2 T_2, \quad (16)$$

where T_1 is the time the main pump will run, and T_2 is the time the slower pump will run, which we know will be the full 4 hours. Also, we know that $R_1 = \frac{1}{2}$ [job/hour] is the rate at which the main pump will run, and $R_2 = \frac{1}{6}$ [job/hour] is the rate at which the slower pump will operate. Substituting

these values into the last equation, we get that

$$1 = \frac{1}{2} T_1 + \frac{1}{6} 4. \tag{17}$$

Solution Part 4.2.2: Solving the problem

Solving this for T_1 we get,

$$T_1 = \frac{2}{3} \text{ hour} = 40 \text{ minutes}. \tag{18}$$

This means that the engineer needs to turn on the main pump after the slower pump has been running for 3 hours and 20 minutes.

9 Word Problem #8.1

In what ratio should water be added to a liquid costing \$12 per liter so as to make a profit of 25% by selling the diluted liquid at \$13.75 per liter?

Solution 8.1.1: Conceptualizing the problem

We'll worry about the ratio after we have calculated how much water should be added to the starting liquid, which we'll set at 1 liter. We lose no generality by doing this. (I'm assuming that the water is free, to keep things simple.)

But first, a word about this 25% profit. How do we deal with it? Percentage changes to a decimal 0.25 as a multiplier. I won't go into detail because I offer this only as a refresher to what the reader is presumed already familiar with.

$$\begin{aligned} \text{(retail cost)} &= \text{(base cost)} + \text{(profit)} \\ &= \text{(base cost)} + (0.25)\text{(base cost)} \\ &= (1.25)\text{(base cost)} \end{aligned}$$

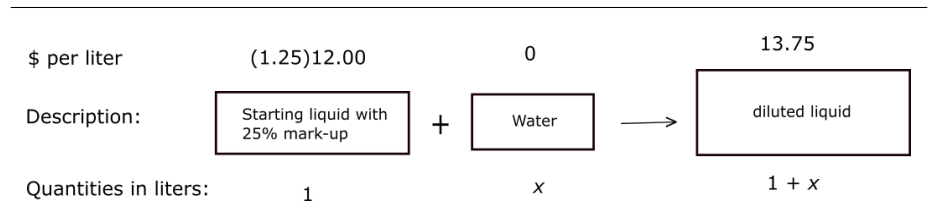


Figure 2. This graphic represents the adding some water x to a starting liquid in a ‘before and after’ process. The arbitrary markup has already been applied to the cost per liter before adding water.

In the graphic in Figure 2, we show a ‘before and after’ process of adding water to this starting liquid. We begin with the conservation equation:

$$\text{(markuped cost of original liquids)} = \text{(required cost of diluted liquid)}. \tag{19}$$

The next adjustment is arbitrary: We are told that the marked-up cost to the consumer will bring in a 25% profit. In other words, the retail cost of the diluted liter of liquid is $(1.25)(\$12.00)$. But the retail cost of the diluted liter of liquid is also given as $(\$13.75)(1 + x)$, from which we get the equation (in dollars)

$$(1.25)(12.00) + 0 \cdot x = (13.75)(1 + x). \quad (20)$$

Solving this, $x = 0.090909\dots$. But we are asked to find the ratio of $x : 1$, which is $0.090909 : 1$, or (approximately) $1 : 11$.

10 Word Problem #8.2

A merchant has 100 lbs of sugar, part of which (x lbs) he sells at 7% profit and the rest (y lbs) at 17% profit. The division of the whole into two parts is to be made so that the net profit is the same as 10% on each original quantity of sugar. How much is each part?

Solution 8.2.1: Conceptualizing the problem

Let's begin with an interpretation that makes sense, at least to me. So, here's my scenario: This merchant is buying 100 pounds of sugar at D dollars per pound wholesale, and will sell it to his retail customers as two different product types, requiring two different packaging schemes, and having two different overhead costs to account for. Part 1 will be a large amount per package, and having a cheap packaging (maybe just dull paper). But Part 2 will be sold in smaller amounts, but packaged in a costlier package, such as a pretty box. The buyers for Part 2 are wealthier than those for Part 1, and are willing to pay a little more. Thus the difference in 'profit' markup per pound accords with the difference in overhead between the two products.

Now, for a figure to help us visualize the data.

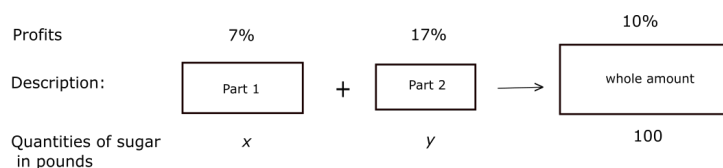


Figure 3. How to divide 100 pounds of sugar to get 10% 'profit'.

So, we have two equations in two unknowns, beginning with the conservation of weight of sugar (in pounds):

$$x + y = 100. \quad (21)$$

And we have the conservation of profit:

$$(\text{profit off of } x) + (\text{profit off of } y) = (\text{net profit off of 100 lbs}). \quad (22)$$

For the next refinement, we'll convert percentages/pound to decimals and multiply rates times quantities off Figure 2, to get

$$.07x + .17y = .10 \cdot 100 = 10.00. \quad (23)$$

Solving (21) and (23) together yields $x = 70$ and $y = 30$ in pounds.

11 Word Problem #8.3

Two vessels A and B contain milk and water in ratios $4 : 3$ and $2 : 3$, respectively. In what ratio should they be added together so that their final mixture is in ratio $1 : 1$?

Solution 8.3.1: Conceptualizing the problem

Notice in Figure 4 that we used arbitrary volume units. One reason for this is that we weren't given a specific unit to work with, and the other is that we can choose any particular unit we please because in taking ratios the volume units will cancel.

milk : water	4 : 3	2 : 3	1 : 1
fraction of milk in total	4 / 7	2 / 5	1 / 2
Description:	A	+	B
	→		final mixture
Quantities in arbitrary volume units:	x		y
			$x + y$

Figure 4. We need to solve for the ratio of x and y .

Now, we've already shown the conservation of volume in the bottom line. We need now only one more equation in x, y to solve for their ratios (hopefully). For that, we show the conservation of milk on both sides.⁷

$$(\text{milk in } A) + (\text{milk in } B) = (\text{milk in final mixture}). \quad (24)$$

Solution 8.3.2: Solving the problem

From (24) we get

$$\frac{4}{7}x + \frac{2}{5}y = \frac{1}{2}(x + y). \quad (25)$$

The variable we need to solve for is x/y , and to do this efficiently, let's divide the last equation through by y , to get

$$\frac{4}{7}x/y + \frac{2}{5} = \frac{1}{2}(x/y + 1). \quad (26)$$

⁷The figure was setup to show the conservation of milk, but we could just as easily have shown the conservation of water.

Let's make one more simplification and substitute $\lambda = x/y$ to get

$$\frac{4}{7}\lambda + \frac{2}{5} = \frac{1}{2}(\lambda + 1), \quad (27)$$

with solution $\lambda = 7/5$. Therefore $x : y :: 7 : 5$.

12 Word Problem #8.4

A can contains a mixture of two liquids A and B in ratio $7 : 5$. After 9 liters are drawn off from the can and replaced by 9 liters of liquid B , the ratio of A to B becomes $7 : 9$. How many liters of liquid A was in the can initially.

Solution Part 8.4.1: Conceptualizing the problem

Let x represent the original total liquid contents of the can. Since we draw off 9 liters and replace it by 9 liters, the final liquid will have x liters in it. Once we determine x , we can then solve for the initial value of A in the can. To simplify the analysis, we'll take as our 'effective' starting condition the state just after the 9 liters of fluid has been drawn off the can.

A : B	7 : 5	0 : 1	7 : 9
Fraction A in total	7 / 12	0 / 1	9 / 16
Description:	Mixture 1 after 9 liters removed	+	B
Quantities in liters:	$x - 9$		x

\longrightarrow

Mixture 2

Figure 5. This graphic represents the adding two mixtures together in a 'before and after' process.

We begin with our usual conservation equation, this time for A .

$$(\text{total } A \text{ in can before adding } B) = (\text{total } A \text{ in can after adding } B). \quad (28)$$

Putting in some details, we get

$$\frac{7}{12}(x - 9) + 0 \cdot 9 = \frac{9}{16}(x). \quad (29)$$

Solution Part 8.4.2: Solving the problem

Therefore the solution for x is 252 liters. And since A was seven-twelfths of x ,

$$\text{initial amount of } A = \frac{7}{12} \cdot 252 \text{ liters} = 147 \text{ liters}. \quad (30)$$

13 Conclusion

A lot of the problems shown above are real-world, but the people who are tasked with solving them aren't math professors. Instead, they are the ordinary low-level IT person whose boss wants to get an estimate on how long it will take two different kinds of printers (working together) to do a rush job. Or, a shopkeeper who needs to know how much mark up on two parts of a 100 pound of sugar is needed to make a reasonable 'profit' off the original purchase, and so on.

I did not include here the oil-to-gas mixture problem, which almost anyone might have to solve to operate a gas-powered machine. (Operating the machine on the wrong ratio can ruin it.) It goes like this. Your gasoline-powered chainsaw requires an oil-to-gas mixture of precise ratio. But you don't have it on hand. Instead, you have on have a mixture that is too high and another that is too low. Question: In what ratio should you mix them to obtain he required mixture ratio?

So, to the student who asks if he or she will ever need algebra in real life, the answers is probably not, but not definitely not. They just might!

Long ago, Sauron fooled us into believing that there are 19 different schemes to rule algebra word problems in the various sciences. Ugh! But we have discovered the truth. There's just One Algebra Scheme to rule them all!