Euclid's Lemma in a Flowchart

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Abstract

Euclid's Lemma is a simple, but beautiful, theorem in number theory. When we view the proof from within a flowchart, it may appear easier to see the heuristics involved in its proof. This proof uses Bézout's Identity.

1 Flowcharting

It's not often that a mathematical proof can be presented in one linear flow from start to end. Often, we will encounter a conditional (or a case statement) that creates parallel flows of logic pathways, forming a tree structure, whose every terminal node is either a contradiction statement or the conclusion we wish to show, in which case, I am likely to terminate it with a **Done** statement in an oval.

Let $P(x)$ be a proposition,[1] maybe dependent on certain variables. The standard flowchart symbol for an if-then-else node is the diamond, as in the following diagram

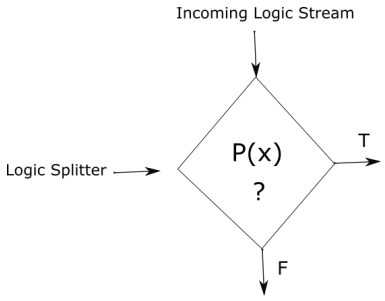

Figure 1. This flowchart fragment reveals the effect of the disjunctive tautology (the Logic Splitter) on the tautology $P \vee \neg P$, being realized as an if-then-else statement. One enters the logic splitter with the tautology $P \vee \neg P$, which contains no information, since it's logically true. However, by exiting the logic splitter by the T-path, we leave it asserting P, but on exiting the logic splitter by the F-path, we leave it asserting $\neg P$.

[1] A *proposition* is a statement that is either true or false.

I call the diamond symbol a *logic splitter* because it bifurcates the logical flow into two distinct directions.

2 Euclid's Lemma

Statement: Let p be any prime number. Let a, b be any two positive integers, neither equal to unity. Claim: If $p \,|\, ab$, then either $p \,|\, a$ or $p \,|\, b$. This seems reasonable, but how to prove it? Note: The symbol '(a, b)' means the greatest common divisor of a and b.

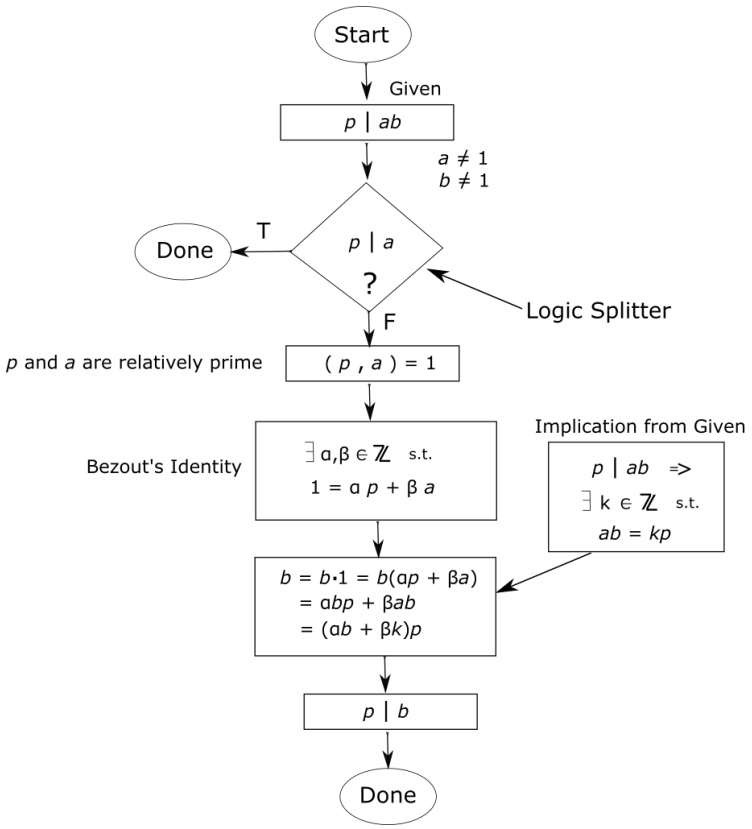

Figure 2. This is a flowchart proof of Euclid's Lemma. The beauty of the Logic Splitter is that either pathway you exit it by, you have more information coming out of it than going into it!