Problems Concerning the GCD

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Abstract

The GCD refers to the *greatest common divisor* of a set of positive integers. The GCD of integers a and b is denoted in this paper as (a, b). The GCD of two integers is the largest (greatest) integer that evenly divides both a and b. Practical uses of the GCD are all over both mathematics and engineering. There are even practical everyday applications for it.

1 Problem 1

Sofia wants to make the largest number of party favors (contents in a bag) for an upcoming party she will host. She wants them to be identical to each other, and each comprise n small boxes of licorice and m candy bars. On hand, she has 21 boxes of licorice and 30 candy bars. What is the greatest number of party favors she can make out of those items?

Let x be the number of party favors to be put together. To divide up the boxes of licorice equally among the party favors, the number of party favors x must divide the number of boxes of licorice. By similar reasoning, x must also divide the number of candy bars. Hence, to make x a common divisor of both and to make this divisor as large as possible, we want

$$x = (21, 30) = 3\,. \tag{1}$$

Thus, to distribute 21 boxes of licorice equally among 3 party favors, requires $21/3 = 7$ boxes per party favor. And to distribute 30 candy bars equally among 3 party favors, requires $30/3 = 10$ candy bars per party favor.

2 Problem 2

Farmer Prescott has a field of dimensions 240 meters by 180 meters. For easier management of the field, he wants to partition the field into square subsections that will completely cover the field. Assuming that the edge dimension of the typical square side will be an integer in meters (no fractional part), what is the fewest number of square subdivisions that will do the job?

To get the fewest number of square subsections, we need the largest side length of the square that will conform to all the constraints given to us. So, to fit the squares into the rectangular field to perfectly fill up each dimension (both length and width), we need this side dimension of the square, call it L, to be as large as possible. Hence, we need L to be the GCD of the given dimensions, that is

$$L = (240, 180) = 60\,. \tag{2}$$

Thus, it will requiire $240/60 = 4$ squares to fill up the long dimension, and it will require $180/60 = 3$ squares to fill up the shorter dimension. That makes for a total of $3 \times 4 = 12$ squares in all, which is the fewest number that will do the job.