Word Problems 12: Mixed-Rate Problems #9

P. Reany

November 22, 2016

Abstract

In this algebra word problem note, we use the Scheme to solve our ninth attempt at what I refer to as a 'mixed-rate problem'. In this type of problem, two or more 'machines' work together at generally different rates to produce subtotals that add to a total. Quantitative information can be given in the problem in various forms: percentages, fractional amounts, or by ratios. We have to know how to deal with each of them.

1 Introduction

The problems here are fairly routine.

2 Word Problem #12.1

A and B together can do a job in 8 days. A and C together can do the job in 9 days. And B and C together can do the job in 10 days. What is B's individual rate?

3 Solution 12.1.1: Conceptualizing the Problem

When two machines, A and B, say, work together over a common time they have a combined effective rate of $\dfrac{1}{T}$, which we get from the equation

$$(R_A + R_B)T = 1[\text{job}]\,, \tag{1}$$

from which we get

$$R_{A+B}^{\text{effective}} = R_A + R_B = \frac{1}{T}\,. \tag{2}$$

Now, on to the rate we need. So, from the given information, we get

$$R_A + R_B = 1/8\,, \tag{3a}$$
$$R_A + R_C = 1/9\,, \tag{3b}$$
$$R_B + R_C = 1/10\,. \tag{3c}$$

According to wolframalpha.com, $R_B = \dfrac{41}{720}$.

4 Word Problem #12.2

A woman sold 100 oranges for $12.10 total. She sold the first kind at the rate of 3 for 35¢ and the second kind at the rate of 7 for 35¢. How many were sold at the first rate?

5 Solution 12.2.1: Conceptualizing the problem

Let's begin with a figure this time.

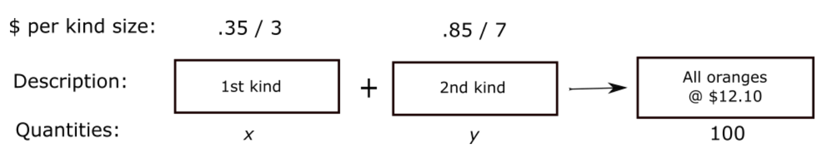

Figure 1. We've converted the cents sign to dollars. Otherwise, the problem is quite familiar to us by now.

Next, we write the familiar 'total as sum of its parts' equation:

$$(\text{Total sales from oranges}) = (\text{sales from 1st kind}) + (\text{sales from 2nd kind})\,. \tag{4}$$

Then, suppressing units, we get

$$12.10 = \frac{.35}{3}\,x + \frac{.85}{7}\,y\,. \tag{5a}$$

We also have the conservation equation on the total number of oranges:

$$100 = x + y\,. \tag{5b}$$

Combining (5a) and (5b), we get the solution for the first kind sold as $x = 9$ oranges.

6 Word Problem #12.3

Question 224473:[1] Three skilled laborers a, b, and c can do a job in 20 days. Just a and b can do the job in 30 days. Just b and c can do the job in 40 days. What are their individual rates in units job/days?

7 Solution 12.3.1: Conceptualizing the Problem

This problem is similar to Problem 12.1 above. We won't need a figure this time, either. Let $x = R_a$, $y = R_b$, $z = R_c$. Then we get the three equations:

$$20(x + y + z) = 1\,, \tag{6a}$$
$$30(x + y) = 1\,, \tag{6b}$$
$$40(y + z) = 1\,. \tag{6c}$$

Solving these together [using wolframalpha.com], we get

$$R_a = 1/40 \text{ [job/day]}\,,$$
$$R_b = 1/120 \text{ [job/day]}\,,$$
$$R_c = 11/60 \text{ [job/day]}\,.$$

8 Word Problem #12.4

A shop keeper wants to make 4 pounds of a tea blended from two ingredients: black tea, costing \$2.20 per pound, and orange pekote tea, costing \$3.00 per pound. If the value of the blended tea is to be \$2.50 per pound, how much of the ingredients are to be use to maintain the value of the ingredients in the blend?

9 Solution 12.4.1: Conceptualizing the Problem

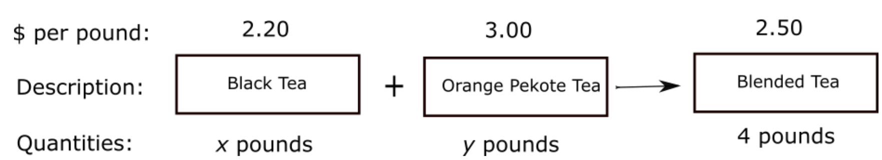

Figure 2. We'll be writing conservation equations on both poundage and cost, or value, on the teas, before and after blending.

[1] Found at https://www.algebra.com/algebra.

10 Solution 12.4.2: Solving the Problem

Now we extract the equations we need.

$$\text{Conservation of poundage:} \qquad x + y = 4\,,$$
$$\text{Conservation of cost (value):} \qquad 2.20x + 3.00y = 2.50(4)\,.$$

The solution for this couple of equations is: $x = 2.5$, and $y = 1.5$, both in pounds, of course.

11 Conclusion

Wolframalpha.com has become my 'graphing calculator.'