Word Problems 13: Mixed-Rate Problems #10

P. Reany

May 16, 2023

Abstract

In this algebra word problem note, we use the Scheme to solve our tenth attempt at what I refer to as a 'mixed-rate problem'. In this type of problem, two or more 'machines' work together at generally different rates to produce subtotals that add to a total. Quantitative information can be given in the problem in various forms: percentages, fractional amounts, or by ratios. We have to know how to deal with each of them.

1 Introduction

Here we introduce a new twist to our problems: a kinematics problem, and in it we will use an inverse rate, just like we used inverses to go from job/time to time/job.

2 Word Problem #13.1

Amanda drove 50 miles. Then she dropped her speed by 20 miles per hour and drove another 5 miles. If the entire trip took one hour and 30 minutes, what was Amand's initial speed?

3 Solution 13.1.1: Conceptualizing the Problem

What are the totals for this problem? Total time and total distance. We'll begin our analysis with total time, but first we nust make some reasonable assuptions.

1. Amanda drove at a constant speed V (in miles per hour) for the first part of her journey.
2. Then she would have driven at the constant speed of $V - 20$ during the second part of her journey.

So now we claim that the total time is the sum of all its parts, which were only two.

$$\text{Total time for jour.} = (\text{time for 1st part of jour.}) + (\text{time for 2nd part of jour.})\,. \quad (1)$$

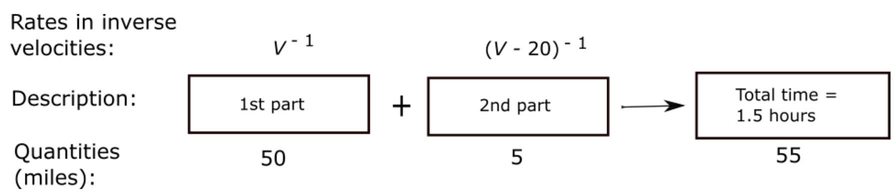

Figure 1. The total time is divided between the time traveled at speed V and the time traveled at speed $V - 20$.

This is a kinematics problem, meaning that it describes motion by four main variables: distance (S), time, velocity, and acceleration, though this last variable won't be needed in this problem. In kinematics, the fundamental equation relating the variables in a constant velocity (speed) problem is this

$$S = VT\,. \tag{2}$$

From which we get

$$T = V^{-1}S\,. \tag{3}$$

Now, we take the total time as the sum of its parts from (1) and from Figure 1:

$$1.5 = V^{-1}(50) + (V - 20)^{-1}(5)\,. \tag{4}$$

According to wolframalpha.com, when I input the line

`‘1.5=V^{-1}(50)+(V-20)^{-1}(5) what is V’`

it returns that V has two possible answers:

$$V = 40 \qquad \text{or} \qquad V = 50/3\,. \tag{5}$$

The latter value is inconsistent with the implied constraint that $V - 20 > 0$. I got better results when I imput the line

`‘Solve for V, [1.5 = V^{-1}(50)+ (V-20)^{-1}(5)]  and [V-20>0]’`

which returned the sole value $V = 40$. Thus the answer to the question is the initial speed is 40 miles per hour.

4 Word Problem #13.2

A chemist needs to make as much 50% acid as he can starting with 5 liters of 70% acid and an unlimited amount of 40% acid (both acids of the same type and percentages are by volume). How much 40% acid should be added to the 70% acid?

5 Solution 13.2.1: Conceptualizing the Problem

Let's begin with a figure.

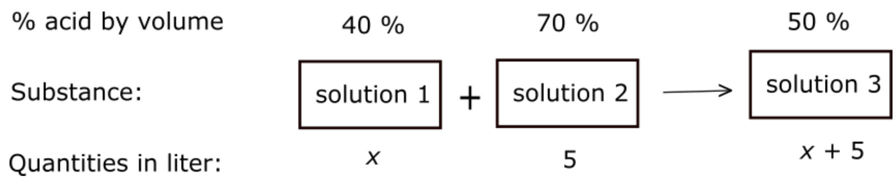

Figure 2. We've already indicated on the figure that the overall volume was conserved. Total acid by volume must be as well.

Next, total acid by volume is conserved:

$$.4(x) + .7(5) = .5(x + 5)\,, \tag{6}$$

which has solution $x = 10$ liters.

6 Word Problem #13.3

Three different varieties of wheat are to be mixed in proportions 1 : 1 : 2 to produce a wheat mixture of specifed requirements. If the cost per kilogram of the first two varieties are, respectively, \$126/Kg and \$135/Kg, and the final mix is \$153/Kg, what is the cost per kilogram of the third variety?

7 Solution 13.3.1: Conceptualizing the Problem

We can satisfy the ratio requirements right off by setting the kilograms of the first variety to be x. Then the next is also x and the third is $2x$. Now, let D be the cost of the third variety in dollars per kilograms.

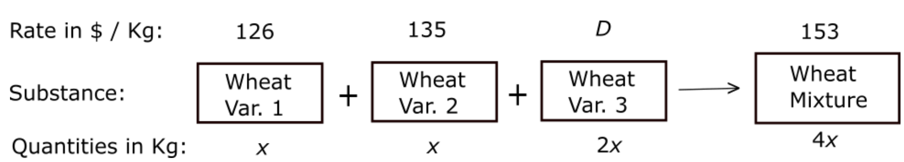

Figure 3. We've already indicated on the figure that the chosen quantities in the diagram satisfy the ratio constraints. Total cost of the ingredients must also be conserved.

The cost conservation equation becomes

$$126(x) + 135(x) + D(2x) = 153(4x)\,. \tag{7}$$

Since the value of x is not zero, we can divide it out of the above equation, to get

$$126 + 135 + D(2) = 153(4)\,, \tag{8}$$

with solution D = $175.50 per kilogram.

8 Word Problem #13.4

Worker A can do a job in 10 days. Worker B can do the same job in 20 days. If A starts the job from the beginning and then alternates days with B, what is the total number of days worked on the job?

9 Solution 13.4.1: Conceptualizing the Problem

We'll make the assumption right off that the number of days worked will be a whole number. Let the number of days A works be N. Then, one of two cases will result. Either 1) A and B will work the same number of days (B works the last day), or 2) A will work one more day than B (A works the last day).

Let's do **Case 1** first.

$$(1/10)N + (1/20)N = 1\,, \tag{9}$$

has solution $N = 20/3$, which is not a whole number. Since that didn't work, we try **Case 2**:

$$(1/10)N + (1/20)(N - 1) = 1\,, \tag{10}$$

which has solution $N = 7$. And that does work. Therefore the total elasped time worked is 7 + 6 = 13 days.

10 Conclusion

Let's make explicit some of the tricks uncovered for solving word problems up to this point:

- Look for totals and parts.
- Look for conserved quantities.
- Make **implicit** assumptions and constraints **explicit**.
- Deal with extraneous roots of quadratic equations.
- If useful, make a figure to organize data.
- Break a process into parts or segments.
- Deal with cases, as in the last problem.