Word Problems 14: Mixed-Rate Problems #11

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June 10, 2021

Abstract

In this algebra word problem note, we use the Scheme to solve our eleventh attempt at what I refer to as a 'mixed-rate problem'. In this type of problem, two or more 'machines' work together at generally different rates to produce subtotals that add to a total. Quantitative information can be given in the problem in various forms: percentages, fractional amounts, or by ratios. We have to know how to deal with each of them.

1 Introduction

Sometimes a figure is the best way to organize data for yourself and for your readers.

2 Word Problem #14.1

A vendor wishes to buy fruit concentrate at \$10 per quart and mix it with water in two different proportions to make two different drinks, at two different sales prices, to make two different profits off each mix. The high concentrate mix is in ratio $1:3$ concentrate to water, and the low concentrate mix is in ratio $1:4$ concentrate to water. The vendor sells the high concentrate for \$6 per quart, and the low concentrate for \$4 per quart. If on a given evening, he sells twice as much of the low concentrate mix as high concentrate mix, what profit does the vendor make per quart?

3 Solution 14.1.1: Conceptualizing the problem

Let's begin with a figure.

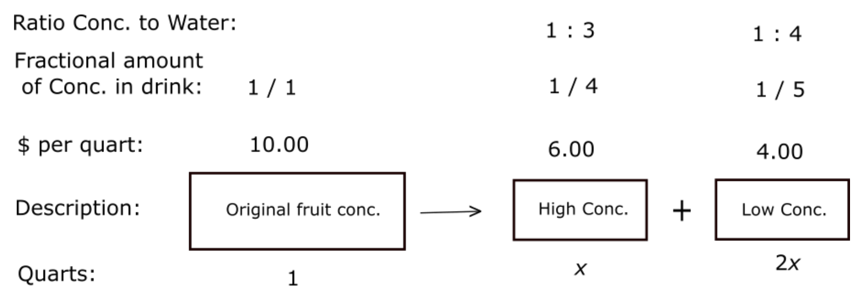

Figure 1. The overall volumes of the mixes are not conserved, but their fractional amounts in the mixes are.

Let's be clear on what we mean by Profit.

$$\text{Profit} = (\text{Sales Price}) - (\text{Cost})\,. \tag{1}$$

Profit per quart of fruit concentrate is no more complicated than to write:

$$\text{Profit/quart} = (\text{Sales Price/quart}) - (\text{Cost/quart})\,. \tag{2}$$

The cost/quart of fruit concentrate is simple. It's just the original \$10.00 because we assume no cost to add all the water we want to make the two mixes. The next equation we can write down is for the conservation of fruit concentrate (with the unit of quarts suppressed)[1]:

$$1 = (1/4)x + (1/5)(2x)\,. \tag{3}$$

which has solution $x = 20/13$ quarts for the sale of the high concentrate. Therefore $2x = 40/13$ quarts for the sales of the low concentrate.

$$\begin{aligned}(\text{Sales Price/quart}) &= 6.00x + 4.00(2x)\\ &= 6.00(20/13) + 4.00(40/13)\\ &\approx 21.54\,. \end{aligned} \tag{4}$$

which is the dollar amount per quart. So, take away the original cost of \$10 per quart, and we get the profit per quart to be \$11.54.

4 Word Problem #14.2

A job takes 4 hours for two people (A and B) to perform. A, working alone, can do the job in 6 hours. How long would it take B to perfom the job alone?

[1] Think of it this way. If I take a pile of nickels and divide it into two piles, I still have the same number of nickels, right? Now, if I add a bunch of pennies to each pile of nickels, I increase the number of coins per pile, but the overall number of nickels has remained the same. In like manner, adding water to each mix changes the volumes of each, but leaves invariant the original amount of fruit concentrate.

5 Solution 14.2.1: Conceptualizing the Problem

Now, total of 1 job is performed by both people:

$$1 \text{ job} = (\text{PJDB } A) + (\text{PJDB } B)\,. \tag{5}$$

Expanding a bit, we get

$$1 = R_A T_A + R_B T_B = (R_A + R_B)T\,, \tag{6}$$

since $T_A = T_B = T$, the common time they work together, which is 4 hours. And $R_A = 6$ hours. Therefore,

$$1 = (\frac{1}{6} + R_B)4\,, \tag{7}$$

which has solution $R_B = \frac{1}{12}$ job/hour.

6 Word Problem #14.3

A pump can fill a tank in 3 hours. This time, the tank a sprung a leak and now it take 3.5 hours to fill the tank. How long does it take the leak to empty the tank?

7 Solution 14.3.1: Conceptualizing the Problem

Let R_P be the rate in job/hours for the pump to fill the tank. Let R_L be the rate in job/hours for the leak to empty the full tank. Since the leak works against filling the tank, R_L gets a negative sign in front of it.

$$R_P T_P - R_L T_L = 1[\text{job}]\,. \tag{8}$$

Substituting in the values we know, we get

$$(\frac{1}{3} - R_L)(3.5) = 1\,. \tag{9}$$

with solution $R_L = \dfrac{1}{21}$ job/hour.

8 Word Problem #14.4

Twenty workers can do a job in 35 days. Beginning with 20 workers, work proceeds as normal for 11 days. After that time, 5 workers quit but are not replaced for the next 4 days. How many workers should be hired on day sixteen so that the job will be finished on the 35th day?

9 Solution 14.4.1: Conceptualizing the Problem

Let's visualize this as a timeline figure.

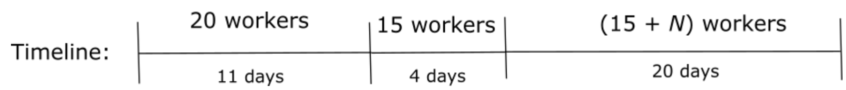

Figure 2. We divide up the whole job into three time periods, each contributing to the whole job.

We'll make the assumption right off that the rate R at which each worker works (in job/day) is the same for all of them. We can easily calculate R from the first information given:

$$R(35 \text{ days}) + R(35 \text{ days}) + \cdots + R(35 \text{ days}) = 1[\text{job}]\,, \tag{10}$$

where there are 20 terms in this sum. Therfore, we get

$$(20R)(35 \text{ days}) = 1[\text{job}]\,, \tag{11}$$

from which we have that $R = \frac{1}{700}$ job/day.

Next, we divide up the whole job into three contributing parts and sum on those three contributions:

$$\begin{aligned} 1 \text{ job} &= (\text{PJDB 1st part}) + (\text{PJDB 2nd part}) + (\text{PJDB 3rd part}) \\ &= \left(20\,\frac{1}{700}\right)(11) + \left(15\,\frac{1}{700}\right)(4) + \left((15+N)\,\frac{1}{700}\right)(20)\,. \end{aligned} \tag{12}$$

Multiplying through by 700, we get

$$700 = (20)11 + (15)4 + (15+N)20\,, \tag{13}$$

which has solution $N = 7$. Meaning, that for the last twenty days of work, seven more workers should be hired to finish the job on the 35th day.

10 Conclusion

Conceptualizing a process in time as distinct, but separately contributing, time segments can be useful in finding the solution — they often become the parts to the whole. This will be especially important when we return to kinematics.