Word Problems 15: Mixed-Rate Problems #12

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Abstract

In this algebra word problem note, we use the Scheme to solve our twelfth attempt at what I refer to as a 'mixed-rate problem'. In this type of problem, two or more 'machines' work together at generally different rates to produce subtotals that add to a total. Quantitative information can be given in the problem in various forms: percentages, fractional amounts, or by ratios. We have to know how to deal with each of them.

1 Introduction

We need to get used to using inverse rates just as easily as regular rates, just like inverting job/time to time/job. In kinematics, we need to get used to using inverse speed (velocity) v^{-1} as in $T = v^{-1}D$.

2 Word Problem #15.1

A widow received $\frac{1}{3}$rd of her husband's estate, and each of her three sons received $\frac{1}{3}$rd of the balance. If the sum the widow received is added to that of one of her sons, the total is \$60,000. What was the value of the estate?

3 Solution 15.1.1: Conceptualizing the Problem

We'll begin by writing that the value of the estate is equal to the sum of all of its distributed parts. Let the value of the estate be E.

$$
\begin{aligned}
E &= (\text{widow's amount}) + (\text{Son 1}) + (\text{Son 2}) + (\text{Son 3}) \\
&= W + S_1 + S_2 + S_3 \\
&= [\,W + S_1\,] + \frac{1}{3}\Big(\frac{2}{3}E\Big) + \frac{1}{3}\Big(\frac{2}{3}E\Big) \\
&= 60,000 + \frac{1}{3}\Big(\frac{2}{3}E\Big) + \frac{1}{3}\Big(\frac{2}{3}E\Big)\,,
\end{aligned}
\tag{1}
$$

which gives the answer $E = \$108{,}000$.

4   Word Problem #15.2

An ultralight plane has been flying for 40 minutes when a change in wind direction doubles its ground speed. If the entire trip of a 160 miles took 2 hours, how far did the plane travel during the first 40 minutes?

5   Solution 15.2.1: Conceptualizing the Problem

We have two totals: the total distance and the total time of flight, both divided into two parts.

$$
\begin{aligned}
160 \text{ mi} &= (\text{distance part 1}) + (\text{distance part 2}) \\
&= D_1 + D_2 \,.
\end{aligned}
\tag{2}
$$

Converting 40 minutes to 2/3 hour, we have

$$
T_1 = V_1^{-1} D_1 = 2/3 \text{ hour} \,,
\tag{3}
$$

therefore,

$$
\begin{aligned}
2 \,[\text{hrs}] &= (\text{time part 1}) + (\text{time part 2}) \\
&= T_1 + V_2^{-1} D_2 \\
&= 2/3 + (2V_1)^{-1}(160 - D_1) \\
&= 2/3[1 - \tfrac{1}{2}] + \tfrac{1}{2}(160)(V_1)^{-1} \\
&= \frac{1}{3} + 80(V_1)^{-1} \,,
\end{aligned}
\tag{4}
$$

with solution $V_1 = 48$ mph.
   Therefore, $D_1 = V_1 T_1 = 48 \cdot 2/3 = 32$ miles.

6   Word Problem #15.3

How many people N can eat a meal containing soup, salad, and spaghetti, given the following contraints:

1. All of the food will be served from from 55 bowls, each containing exactly one of soup, salad, or spaghetti.
2. Each person gets his or her own bowl of soup.
3. When the people are paired off, each pair gets only one bowl of spaghetti.
4. When the people are tripled off, each triple gets only one bowl of salad.

7 Solution 15.3.1: Conceptualizing the Problem

Every one of the 55 bowls will contain exactly one of soup, salad, or spaghetti. Therefore,

$$55 \text{ bowls} = (\text{bowls of soup}) + (\text{bowls of spagetti}) + (\text{bowls of salad}) \,. \quad (5)$$

Substituting into this, we get

$$55 \text{ bowls} = N + \frac{1}{2} N + \frac{1}{3} N \,, \quad (6)$$

which has solution $N = 30$.

8 Word Problem #15.4

Ralph scored 61 points on a 10-question test that scores +10 points for every correct answer and -3 points for every incorrect answer. How many questions did Ralph get correct?

9 Solution 15.4.1: Conceptualizing the Problem
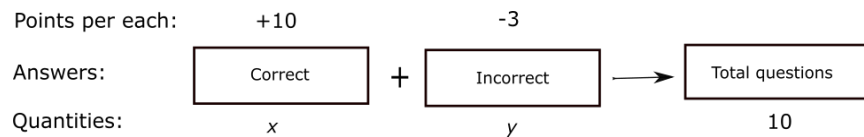
Let's use a figure for this one.

Figure 1.Test scores added: The positive and the negative.

Obviously,
$$x + y = 10 \,. \quad (7)$$

And, the total points are given as the sum of rate times quantity for both correct and incorrect answers:
$$(+10)x + (-3)y = 61 \,. \quad (8)$$

The solution to (7) and (8) is $x = 7$ and $y = 3$. Ralph got seven questions correct.

10 Conclusion

It pays to study hard!