Word Problems 16: Mixed-Rate Problems #13

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Abstract

In this algebra word problem note, we use the Scheme to solve our thirteenth attempt at what I refer to as a 'mixed-rate problem'.

1 Introduction

Some tricky problems this time.

2 Word Problem #16.1

A heat-loss survey by an electrical company indicated that a wall of a house containing 40 ft^2 of glass and 60 ft^2 of plaster lost 1920 BTU of heat (in a given time period). A second wall containing 10 ft^2 of glass and 100 ft^2 of plasteer lost 1160 BTU of heat. Determine the heat lost per square foot of glass and plaster in that house. This problem comes from [1].

3 Solution 16.1.1: Conceptualizing the Problem

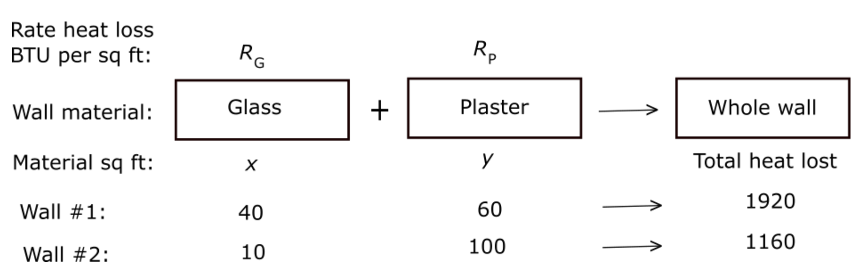

Figure 1. Heat leakage through glass and plaster.

Somehow this clever heat-loss technician is able to measure the heat lost through an entire wall. He then measures the square footage of the glass and plaster of this wall, and repeats for another wall, and then uses algebra to infer the heat loss through just the glass or just the plaster.

We can do this ourselves. The total heat lost for both walls is equal to the sum of the heats lost through their glass parts and their plaster parts:

$$\begin{aligned} 1920 &= 40x + 60y\,, \\ 1160 &= 10x + 100y\,, \end{aligned} \tag{1}$$

where $x = R_G$ and $y = R_P$. This makes it easier to copy the text into the solver, which gives back $x = R_G = 36$ [BTU] and $y = R_P = 8$ [BTU].

4 Word Problem #16.2

Nine liters are drawn from a tank full of wine. Then 9 liters of pure water are added to the tank and the mix is allowed to homogenize. After that, 9 more liters are drawn off and again replaced by 9 liters of pure water and allowed to homogenize. If the final wine-to-water mix is in ratio 16:9, how much does the full tank hold?

5 Solution 16.2.1: Conceptualizing the Problem

We already did a similar problem in Word Problems #8.4. We can solve this problem in two steps. First, let x be the full volume of the tank. Second, after replacing 9 liters of the orignal wine by 9 liters of water, we end up with Mix 1 with wine-to-water ratio $(x - 9) : 9$ and fraction of wine-to-total mixture $(x - 9) : x$ (no figure for this point in the analysis).

Next, we draw off 9 more liters of this mixture and that takes us to Step 2, the setup in Figure 1.

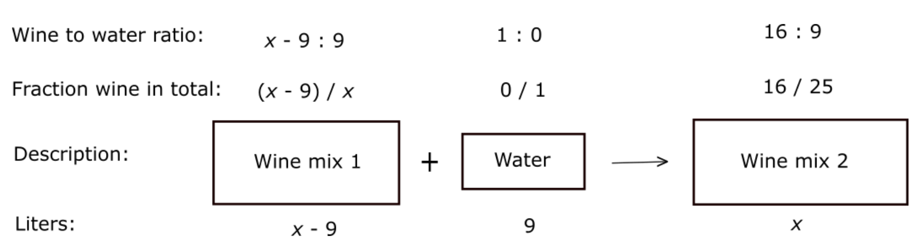

Figure 2. Two-step analysis will do the trick.

The process of adding water to Mixture 1 in Figure 2 tells us that the total amount of wine is conserved in this process:

$$\frac{x-9}{x}\,(x-9) + 0(9) = \frac{16}{25}\,(x)\,, \tag{2}$$

which has solution $x = 45$ liters.[1]

6 Word Problem #16.3

Question 147255.[2] A distilate flows into an empty 64-gallon drum at spout A and out of the drum at spout B. If the influx at A is 2 gallons per hour, what is the outflux rate at B so that the drum is full in 96 hours?

7 Solution 16.3.1: Conceptualizing the Problem

This is a simple 'total is the sum of parts', except that that outflux rate gets a minus sign.

$$64 \text{ gallons} = (\text{contribution at } A) - (\text{contrary outflux at } B)\,. \tag{3}$$

$$64 \text{ gallons} = R_A T - R_B T = (R_A - R_B)T = (2 - R_B)96\,, \tag{4}$$

which as solution $R_B = 4/3$ gallons/hour.

8 Word Problem #16.4

A woman must control her diet. She selects milk and bagel for breakfast. How much of each should she serve in order to consume 700 calories and 28 grams of protein? Each cup of milk contains 170 calories and 8 grams of protein. Each bagel contains 138 calories and 4 grams of protein. (Problem found in [2].)

9 Solution 16.4.1: Conceptualizing the Problem

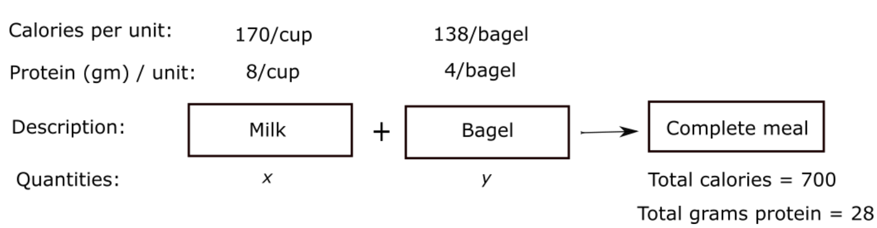

Figure 3. As usual, rates are placed above and quantities and totals are placed below.

We simply have two totals to deal with as constraints on the amounts of each. Referencing Figure 3, we have that

$$\begin{aligned} 170x + 138y &= 700\,, \\ 8x + 4y &= 28\,, \end{aligned} \tag{5}$$

[1]The possible root $x = 5$ is unphysical because it does not satisfy the constraint $x - 9 > 0$.
[2]Found at https://www.algebra.com/algebra.

which has solution

$$x = \frac{133}{53} \approx 2.5 \quad \text{and} \quad y = \frac{105}{53} \approx 2\,. \tag{6}$$

That is, the meal is to consist of 2.5 cups of milk and 2 bagels.

10 Conclusion

To solve tricky problems, it helps to know the available tricks.

References

[1] R. Blitzer. *Intermediate Algebra for College Students*, 3rd Ed. Prentice-Hall (2002), p. 169.

[2] H. Rolf. *Finite Mathematics*, 5th Ed. Brooks/Cole (2002), p. 57.