Word Problems 20: Mixed-Rate Problems #17

P. Reany

December 4, 2016

Abstract

In this algebra word problem note, we use the Scheme to solve our seventeenth attempt at what I refer to as a 'mixed-rate problem'.

1 Introduction

In this note, we're leaving all ambiguities regarding percentages behind us for a while, and begin with a fun problem: The dehydrating watermelon.

2 Word Problem #20.1

This problem brings us some unituitive results: A 100Kg watermelon is estimated to be 99% water and 1% flesh part (that is, what would be left of the watermelon if all water were removed from it). After some days, the watermelon has dehydrated a bit and its water content is down to 98%. How much does the dehydrated watermelon weigh?

3 Solution 20.1.1: Conceptualizing the Problem

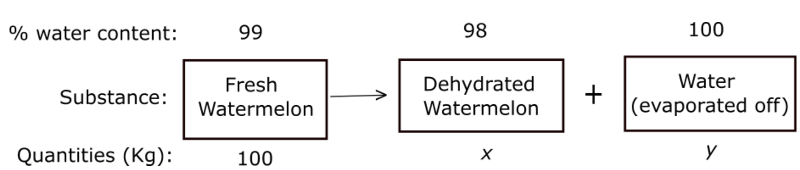

Figure 1. Is x close to the original weight or not?

To properly analyze this thought experiment, it's crucial to think of the watermelon as being made up of two parts of stuff: 'watermelon flesh' and water. Both of these are conserved in this process. Though we won't consider capturing the evaporated water, it still exists in the atmosphere.

We could solve for x by balancing on water in this 'before and after' rocess, but it's even easier to balance on the watermelon flesh part. In the 'before' watermelon, the flesh part constitues 1%, and in the 'after' part it constitues 2%:

$$.01(100) = .02x\,, \tag{1}$$

which has the solution $x = 50\,\text{Kg}$.

4 Word Problem #20.2

Question 835994:[1] To produce sausage formulations, packers normally use lean beef, pork belly, and soy concentrate. The manufacturer uses 3 percent soy in final sausage mass. Water is added to the formulation as ice. The final sausage must contain: 15 percent protein, 60 percent moisture, and 25 percent fat. The ingredients contain the following:

- `lean beef  -- 20% protein, 67% moisture, 13% fat`
- `pork belly -- 10% protein, 40% moisture, 50% fat`
- `soy        -- 90% protein,  7% moisture,  3% fat`

How much of each ingredient should be combined to make 600 kg of sausage emulsion?

Note: The value of the fat content in the soy was written as 0%, but that didn't make 100% when adding up all three components, so I changed it accordingly.

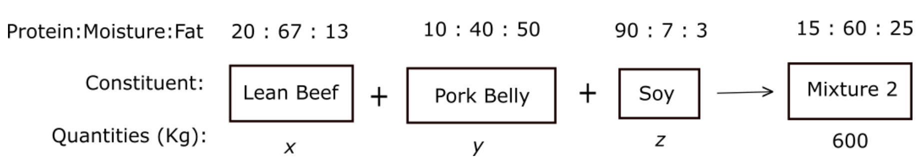

Figure 2. Is there a solution for x, y, and z?

5 Solution 20.2.1: Conceptualizing the Problem

We begin by making use of the information that 3% of the total mix is the soy content (in the 'after' state). Therefore,

$$z = (.03)(600) = 18\,. \tag{2}$$

So, the soy-reduced mass equation becomes

$$x + y = 582\,. \tag{3a}$$

[1]Found at https://www.algebra.com/algebra.

We need only one more equation to solve for x and y. Let's choose balancing on protein:

$$(.20)x + (.10)y + (.90)(18) = (.15)600\,. \tag{3b}$$

Wolframalpha.com gives as solutions for (3a) and (3b) as $x = 156$ and $y = 426$.

6 Word Problem #20.3

A jar containing a mixture (Mix 1) of two liquids, A and B, in ratio $A : B :: 4 : 1$. If 10 liters is removed and replaced by 10 liters of B, the ratio of A to B becomes $2 : 3$. How much of A was in Mix 1?

7 Solution 20.3.1: Conceptualizing the Problem

The initial volume of Mix 1 (call it x) is the same as the final volume of Mix 2.

And accompanying figure is, please:

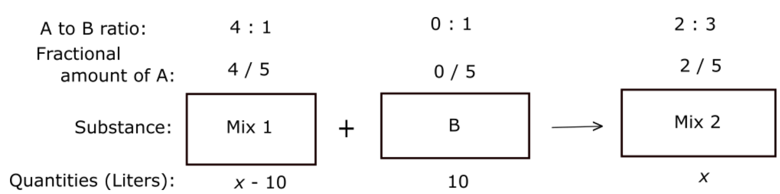

Figure 3. Only one unknown means we need only one equation. To get it, we can choose to balance on A or on B. Which should we pick? (Perhaps you were expecting to see 0/1 over as the 'Fractional amount of A' in B, and that would be correct, but 0/5 is also correct but also has the same denominator as the other two fractions.)

We need only one equation to derive x from. Let's use the conservation of A equation.

$$\frac{4}{5}(x - 10) + 0(10) = \frac{2}{5}(x)\,, \tag{4}$$

which has solution $x = 20$. Thus, the fractional amount of A is Mix 1 is 4/5 th that, being 16 liters.

8 Word Problem #20.4

Question 10885:[2] A car gets 28 mpg on highway, 22 mpg in city. If the total trip is 627 miles using 24 gallons of gas, how many miles were driven in city?

[2] Found at https://www.algebra.com/algebra.

9 Solution 20.4.1: Conceptualizing the Problem

Another easy problem once we identify the totals and their parts. To use the information that the total gasoline consumed on the trip was 24 gallons, we need to write the rates above the rectangles as gallons per mile.

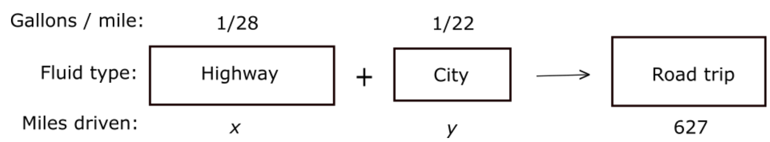

Figure 4. We've inverted miles per gallon to get the more useful conversion rate of gallons per mile.

For the total miles driven (on highway + in city) we get

$$x + y = 627\,. \tag{5}$$

For the total gallons of gasoline consumed, we get:

$$\frac{1}{28}\,x + \frac{1}{22}\,y = 24\,. \tag{6}$$

These last two equations have solutions $x = 462$ miles and $y = 165$ miles, the latter being the miles driven in city.

10 Conclusion

Once you've setup the figure properly, getting a system of equations to solve should be straightforword.