Word Problems 21: Mixed-Rate Problems #18

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Abstract

In this algebra word problem note, we use the Scheme to solve our eighteenth attempt at what I refer to as a 'mixed-rate problem'.

1 Introduction

This time, we have some more ambitious problems to solve.

2 Word Problem #21.1

Question 364411:[1] Two alloys contain silver and copper in the ratios 3 : 1 and 5 : 3, respectively. The alloys are mixed to get a third alloy. The possible ratio of silver to copper in the third alloy is?

3 Solution 21.1.1: Conceptualizing the Problem

First, I believe that the question given us in the problem is ill-posed. It should state: Determine the ratio of silver to copper in the final alloy as a function of the arbitrary amounts of the two alloys mixed together.

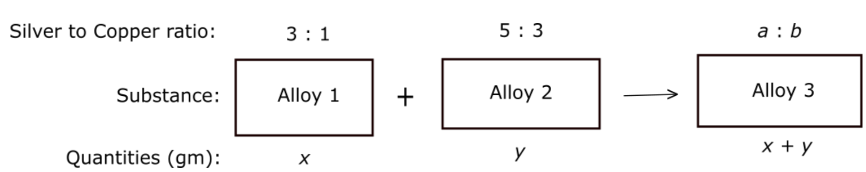

Figure 1. The value we need to solve for is a/b.

Second, I've made the assumption that the ratios were given by weight (or mass). I've chosen grams, but any unit of weight (mass) will do because in the ratio we're looking for, the units will cancel out.

[1]Found at https://www.algebra.com/algebra.

Notice that the figure reflects the fact that the problem as stated makes no restriction on the relative amounts of silver and copper that can be mixed together. Therefore, it's reasonable to assume right now that x and y variables are independent and that they will show up in the answer on an equal footing.

Now, to get the ratio of a to b, we begin by balancing on both the silver and copper components of the process.

$$\text{Balance on silver:} \qquad \frac{3}{4}\,x + \frac{5}{8}\,y = \frac{a}{a+b}\,(x+y)\,, \tag{1a}$$

$$\text{Balance on copper:} \qquad \frac{1}{4}\,x + \frac{3}{8}\,y = \frac{b}{a+b}\,(x+y)\,. \tag{1b}$$

Now, we let $\lambda = a/b$, as we have done in the past. Then we take (1a) $\div$ (1b), and simplify to get

$$\frac{\frac{3}{4}\,x + \frac{5}{8}\,y}{\frac{1}{4}\,x + \frac{3}{8}\,y} = \lambda\,. \tag{2}$$

One last simplification gives us

$$\lambda = \frac{6x+5y}{2x+3y}\,. \tag{3}$$

Let's go one step further and ask ourselves how much copper should be added to 100 grams of silver to produce an alloy of quality sterling silver? First, we need to know that sterling silver has quality defined to be 92.5% silver against all other metals, in this case copper.

Plugging-in the values into (3), we get $\lambda = 92.5/7.5$, so

$$92.5/7.5 = \frac{600+5y}{200+3y}\,. \tag{4}$$

which has the unrealistic solution $y = -58.3333$. This means that we'll never get sterling silver by this combination of alloys, and it suggests that we have tried an out-of-bounds value for λ. Let's investigate this.

If we take $y = 0$, what λ do we get? We get, $\lambda = 3/1$, yielding a percentage of 75%. If we take $x = 0$, what λ do we get? We get, $\lambda = 5/3$, yielding a percentage of 62.5%. Therefore, it's reasonable to believe that the λ values will vary from a low of $5/3$ to a high of $3/1$. Let's now prove this.

With just a little effort, Equation (3) can be rewritten as

$$(3\lambda - 5)y = (6 - 2\lambda)x\,. \tag{5}$$

Neither x nor y can be negative, and, since we've already tested what happens when either one is zero, that leaves us with the constraint on their coefficients that they be both positive or both negative.

Case 1) Both positive: $(3\lambda - 5) > 0$ and $(6 - 2\lambda) > 0$. This gives solutions for λ given by

$$\frac{5}{3} < \lambda < \frac{3}{1}\,. \tag{6}$$

Case 2) Both negative: $(3\lambda - 5) < 0$ and $(6 - 2\lambda) < 0$. This gives no solutions for λ.

4 Word Problem #21.2

Question 377832:[2] A bag of peanuts is worth \$0.28 less than the same size bag of cashews. Equal amounts of peanuts and cashews are used to make 45 bags of a mixture that sells for \$1.25 per bag. How much is a bag of cashews worth? (Give your answer to the nearest cent.)

5 Solution 21.2.1: Conceptualizing the Problem

We'll begin by providing symbols for the price per bag of peanuts and cashews, being R_p and R_c, respectively. But we have been given that $R_p = R_c - .28$ (in dollars). And now the figure.

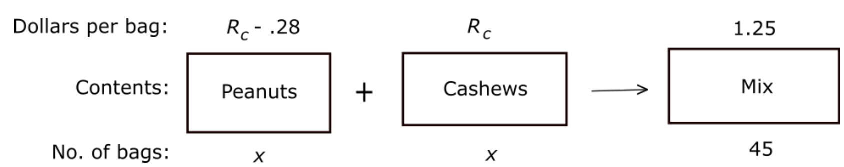

Figure 2. We need two equations to solve for is x and R_c.

Now, we balance on overall number of bags:

$$x + x = 45\,. \tag{7a}$$

And on the overall dollars from the unmixed nuts to the mixed nuts:

$$(R_c - .28)x + (R_c)x = (1.25)45\,. \tag{7b}$$

Wolframalpha.com gives as solutions for (7a) and (7b) as $R_c = \$1.39$.

[2]Ibid.

6 Word Problem #21.3

Question 353765:[3] ....Lead....Zinc....Copper
Alloy A..40%...30%...30%
Alloy B..20%...30%...50%
Alloy C..........10%...90%
How many grams of each alloys A, B, and C must be mixed to get 325 gm of an alloy that is 25% lead, 13% zinc, and 62% copper? I've tried this using the Matrix, and the inverse Matrix, and keep coming up with negitives on some alloys. Thanks for your help.

7 Solution 21.3.1: Conceptualizing the Problem

First, I also got negative numbers with the original data set, so (after some trial and error) I changed the percentages in the final alloy to the values in the table below:

	Lead	Zinc	Copper
Alloy A	40%	30%	30%
Alloy B	20%	30%	50%
Alloy C	0%	10%	90%
Alloy D	22%	25%	53%

Time to put the data into a figure. Note that the percentage information was coded in the form of ratios.

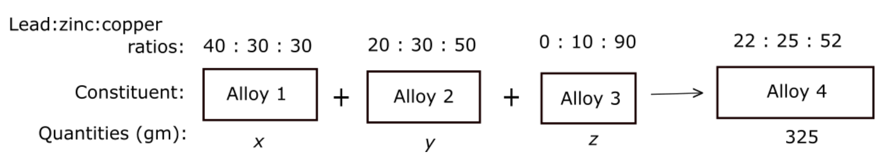

Figure 3. The percentage data is easier displayed as ratios.

Now, balancing on lead, zinc, and copper, in that order, we get

$$40x + 20y + 0z = 22(325)\,, \tag{8a}$$
$$30x + 30y + 10z = 25(325)\,, \tag{8b}$$
$$30x + 50y + 90z = 53(325)\,. \tag{8c}$$

Wolframalpha.com gives as solutions for this system

$$x = 113.75\,, \quad y = 130\,, \quad z = 81.25\,, \tag{9}$$

and these three values add to 325, as they should.

[3]Ibid.

8 Word Problem #21.4

Question 732982:[4] fred is analyzing the cost of producing two different items at an electronics company. an electrical sensing device uses 5 grams of copper and requires 3 hours to assemble. a smaller sensing device made by the same company uses 4 grams of copper but requires 5 hours to assemble. the first device has a production cost of $27. the second device has a production cost of $32. how much does it cost the company for a gram of this type of copper? what is the hourly labor cost at this company? (assuming that production cost is obtained by adding the copper cost and the labor cost)

9 Solution 21.4.1: Conceptualizing the Problem

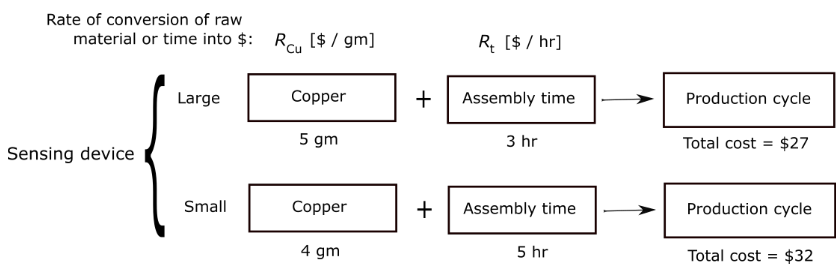

Figure 4. A somewhat different figure than similar problems of this type we've encountered before.

Now, we have one equation each for the total cost being the sum of the component costs for the large and small devices:

$$\text{Balance cost on large device:} \quad R_{\text{Cu}}5 + R_t3 = 27\,, \tag{10a}$$

$$\text{Balance cost on small device:} \quad R_{\text{Cu}}4 + R_t5 = 32\,. \tag{10b}$$

This pair of equations has solution $R_{\text{Cu}} = 3$ and $R_t = 4$. That's $3 per gram of this expensive type of copper.

10 Conclusion

Once you've setup the figure properly, getting a system of equations to solve should be straightforword.

[4]Ibid.