Word Problems 22: Mixed-Rate Problems #19

P. Reany

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Abstract

In this algebra word problem note, we use the Scheme to solve our nineteenth attempt at what I refer to as a 'mixed-rate problem'.

1 Introduction

Getting good at solving problem in a specific domain requires one to invent or borrow a domain-specific problem-solving strategy. For me, this usually begins with a conceptual framework that starts at the big picture, so to speak, and works its way down to the special cases. But how do I know if my efforts are being successful? They are, if all the problems in the domain are beginning to 'look alike', no matter how different they appear superficially.

2 Word Problem #22.1

Question 842253:[1] A reservoir containing 1 million gallons of water has been tainted with arsenic. Scientists conducted several tests and found that the solution in the reservoir is 2% arsenic. Although scientists realize that the arsenic can't be removed from the supply, they do understand that they can reduce the percentage to safe levels. A safe level of arsenic is .001% How much water must be added to the reservoir to make the water safe to drink?

3 Solution 22.1.1: Conceptualizing the Problem

This problem is rather easy, but a bit interesting. (Figure on following page.)

Balance on arsenic:

$$2 \times 10^6 = 0.001(10^6 + x)\,, \tag{1}$$

which has the solution $x \approx 2$ billion gallons.

[1] Found at https://www.algebra.com/algebra.

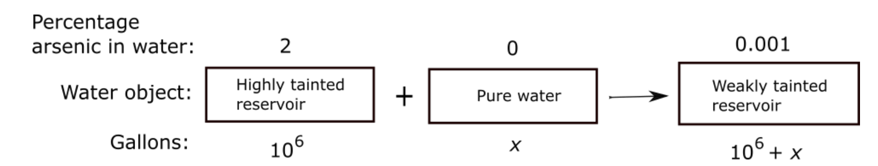

Figure 1. We'll have to add a lot more than a million gallons of water.

4 Word Problem #22.2

Question 722538:[2] Jessica has money in two savings accounts. One rate is 10% and the other is 15%. If she has \$450 more in the 15% account and the total interest is \$258, how much is invested in each savings account?

5 Solution 22.2.1: Conceptualizing the Problem

We haven't been told what kind of interest we are dealing with here, so I will assume that the interest is accrued at the end of a single interest-payment cycle.

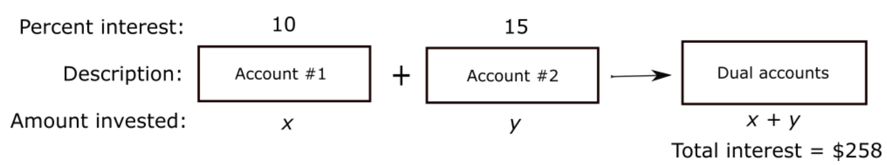

Figure 2. We've also been given the constitutive relation: $y = x + 450$.

Balancing on interest, we have

$$.10x + .15y = 258\,. \tag{2}$$

We also have the constitutive relation: $y = x + 450$. Solving now for x and y, we get

$$x = \$762\,, \quad y = \$1212\,. \tag{3}$$

[2]Ibid.

6 Word Problem #22.3

Question 638401:[3] #6. A textile company has specific dyeing and drying times for its different cloths. A roll of Cloth A requires 65 minutes of dyeing time and 50 minutes of drying time. A roll of Cloth B requires 55 minutes of dyeing time and 30 minutes of drying time. The production division allocates 2440 minutes of dyeing time and 1680 minutes of drying time for the week. How many rolls of each cloth can be dyed and dried?

7 Solution 22.3.1: Conceptualizing the Problem

Apparently in this textile facility, cloths are dyed consecutively and dried consecutively. Therefore their dyeing times and drying times add arithmetically in each category.

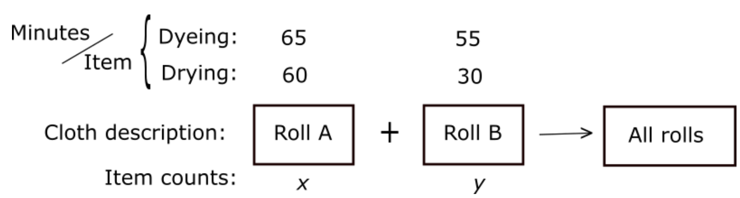

Figure 3. Tables are fine to organize raw data, but a well-thoughtout figure tells a better story of what's going on.

The total minutes allocated for dyeing is given as the sum of its parts:

$$65x + 55y = 2440\,. \tag{4a}$$

The total minutes allocated for drying is given as the sum of its parts, too:

$$50x + 30y = 1680\,. \tag{4b}$$

The solution to x and y from these last two equations yields

$$x = 24\,\text{rolls of Cloth A}\,, \qquad y = 16\,\text{rolls of Cloth B}\,. \tag{5}$$

8 Word Problem #22.4

Question 269702:[4] The time it takes to do homework includes a fixed amount of time to prepare plus a constant amount of time per problem. If a student can do 5 homework problems in 40 minutes, and 10 problems in 70 minutes, how many minutes will 25 problems take?

[3] Ibid.
[4] Ibid.

9 Solution 22.4.1: Conceptualizing the Problem

What makes this problem different than most (maybe all) that we've encountered previously is that the parts of the total seem to enter into the problem as components on very different footings: The fixed time τ enters the figure as a time, not as a count, like its comrade x on the same line. Hence, we get the formula

$$\tau + R_P x = T(x)\,. \tag{6}$$

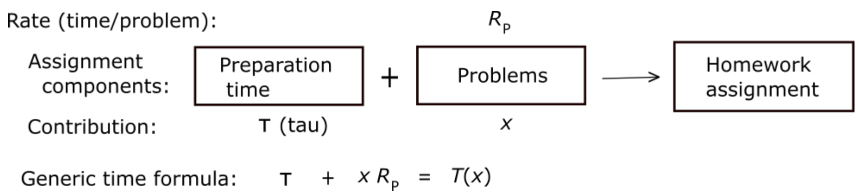

Figure 4. Preparation time τ and problem time enter on a different footing. Note: $R_F(x)$ means R_F times x, whereas $T(x)$ means T is a function of x.

However, we could give 'preparation time' its own conversion factor $R_F = \tau$ minutes/assignment, where the F stands for 'fixed time'. Thus, $\tau = R_F \times 1$ [preparation]. Having done this, we could rewrite (6) as

$$R_F(1) + R_P x = T(x)\,. \tag{7}$$

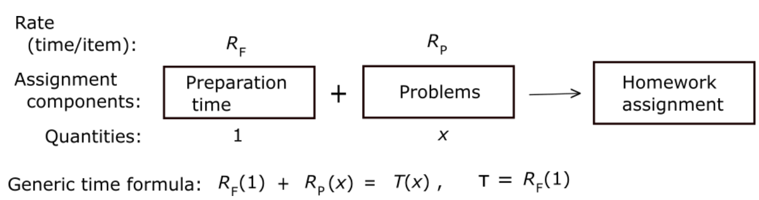

Figure 5. Preparation time τ and problem time enter on a different footing, but this time, however, the figure is now in standard form. Note: $R_F(x)$ means R_F times x, whereas $T(x)$ means that T is a function of x.

It's an interesting side point that if our generic 'student' messedup on his preparation and needed to do it over again, then the count for it would be 2 under the 'Preparation time' rectangle in Figure 5. If he messedup the prep time n times, we could account for this by altering Equation (7) to get

$$R_F \cdot n + R_P \cdot x = T(n, x)\,. \tag{8a}$$

This can be put in the standard form (where explicit referencing of the total's functional dependence is suppressed) as

$$R_F \cdot n + R_P \cdot x = T\,. \tag{8b}$$

We can visualize this generalization of the problem in the figure below.

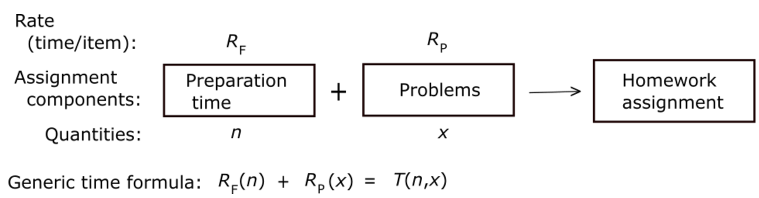

Figure 6. We have finally placed the figure in a truly general standard form by placing an unknown n beneath the 'Preparation time' rectangle.

By the way, we are told that this generic student is able to work each problem at a constant rate. This seems unlikely. So, even though the math doesn't need a re-interpretation of this time unit, commonsense does. Let's interpret this fixed time per problem as an average.

Anyway, from the information given, we can write the coupled equations:

$$\tau + R_P(5) = 40\,, \tag{9a}$$

$$\tau + R_P(10) = 70\,. \tag{9b}$$

The solution for R_P is 6 minutes/problem, and for τ is 10 minutes/assignment.

10 Conclusion

It might seem to the experienced problem solver that I have belabored minor issues in the last problem, but I disagree. Conceptualizing the problem is a very different, and usually more difficult, mental activity than solving the problem. For example, by the time we derive the system of equations (9a) and (9b), the solution is trivial (by hand or by computer). It's the conceptualization of the problem where the real challenge (and real fun) lies. And it's where most inexperienced students get stuck. They evidence this stuckness by saying, "I don't even know how to get started!"

Friends, this shouldn't be the case! The general mode of heuristics for solving problems in a given category (in this case the 'mixed-rate' problems) is to build for yourself a conceptual model abstracted from the problems you've solved up to that point, and make it general enough so that every new problem, no matter how nonconformist it may appear outwardly, is revealed, with some analysis, to be just another example of your general form. This is why I have from the start emphasized conceptualizing the problem rather than the perfunctory matter of 'solving' the problem. This is also why I have a strong aversion to formulating the solution to a problem in tabular form. I have no problem providing raw data in tabular form, but modeling the entire problem in tabular form is both repulsive and unrevealing to me.

The reason why the last problem was so interesting to me was because it challenged me to find the general model to which is was a specific instance. And, by this point in the series, we're well familiar with the general form. The general model for the last problem is seen in Figure 6, but this generality was hidden by problem having given us that n was set equal to 1 from the start. Therefore, the whole structure of 'rate times quantity' was, for this component term, hidden, or rather implied. This hiddenness tended to make the problem look different from what we've encountered before.

Question: How do you know when your conceptual model you use to solve problems in a given category is general enough? Answer: When all the problems you've encountered, and continue to encounter, start to **look the same** to you!