Word Problems 23: Mixed-Rate Problems #20

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Abstract

In this algebra word problem note, we use the Scheme to solve our twentieth attempt at what I refer to as a 'mixed-rate problem'.

1 Introduction

Percentage problems with even a few constitutive relationships can get algebraically messy.

2 Word Problem #23.1

Question 286358:[1] An urn is filled with coins and beads. All the coins are either silver or gold. Twenty percent of the objects in the urn are beads. Forty percent of the coins in the urn are silver. What percent of the objects in the urn are gold coins?

3 Solution 23.1.1: Conceptualizing the Problem

The parts are beads, silver coins, and gold coins. Cue the figure.

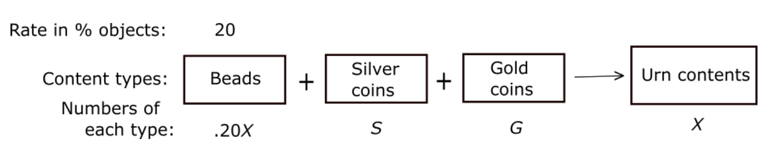

Figure 1. We have to deal with a few constitutive relations this time.

We are asked to find the percentage of gold coins in the urn (compared all objects in the urn), which is given by

$$P = \frac{G}{X} \times 100\% \,, \tag{1}$$

[1]Found at https://www.algebra.com/algebra.

where X be the sum of all the objects in the urn.

4 Solution 23.2.1: Solving the Problem

This is one of those special problems worthy of having the solution presented in detail.

The overall total gives us (for beads, silver coins, and gold coins)

$$X = B + S + G\,. \tag{2}$$

But B is 20% of X, leaving $0.80X = S + G$, or

$$0.80X = C\,, \tag{3}$$

where C stand for the number of all coins. Since the silver coins are 40% of all coins, the gold coins are 60%.

$$.60C = G\,. \tag{4}$$

Using (3) and (4) we can write a simple relation between X and G: Multiply (3) by 0.60 and use (4) to get

$$.60(.80)X = G\,, \tag{5}$$

which gives us that G takes 48% of the total contents of the urn.

5 Word Problem #23.2

Question 255570:[2] It is required to make 12 grams of certain chemical compound called Z. This is made from compounds W, X, and Y in the ratio of 2:1:3. The compound Y is itself made from W and X. To make 6 grams of Y requires 4 grams of W and 2 grams of X. How much W and X is required to make the required amount of Z.

6 Solution 23.2.1: Conceptualizing the Problem

We need to make 12gm of compound Z using W,X,Y in ratios 2:1:3. Taking the total as the sum of its parts, we write

$$2t + t + 3t = 12\,. \tag{6}$$

From this we get

$$t = 2[\text{grams}]\,. \tag{7}$$

Let's make a figure from this.

[2] Ibid.

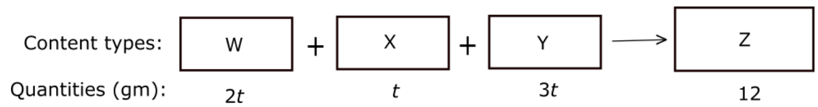

Figure 2. The quantities of W, X, and Y have been chosen to enforce the ratios 2:1:3.

And from this we know that there are 6 grams of Y. But we are given that 6 grams of Y contains 4 grams of W and 2 grams of X. So, we add these quantities to the W and X in the figure, to get $2t + 4$ for W and $t + 2$ for X. Therefore,

$$W = 8[\text{grams}]\,, \quad X = 4[\text{grams}]\,. \tag{8}$$

7 Word Problem #23.3

Question 154928:[3] I have 51 handle bars and 116 wheels. Using all the parts, how many tricycles and bicycles can I assemble?

8 Solution 22.3.1: Conceptualizing the Problem

Each bicycle and each tricycle will dibs certain necessary parts. For example, they'll all dibs one handle bar. They'll each dibs wheels as appropriate to their design requirements. Let's make a figure.

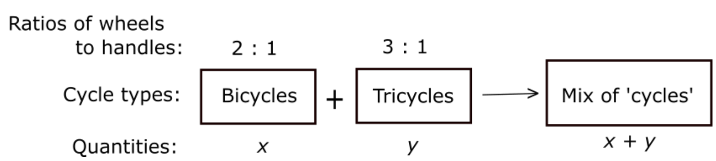

Figure 3. I didn't choose to represent the data as ratios to be novel or cute. They're simply an efficient way to represent the data.

Let's balance on handle bars first.

$$\frac{1 \text{ handle bar}}{\text{bicycle}}\, x + \frac{1 \text{ handle bar}}{\text{tricycle}}\, y = 51\,. \tag{9}$$

This simplifies to

$$x + y = 51\,, \tag{10a}$$

[3]Ibid.

proving that the $x+y$ in Figure 3 is 51. Next, let's balance on wheels, only let's suppress the units this time.

$$2x + 3y = 116\,. \tag{10b}$$

If we can get positive integer solutions for (10a) and (10b), then we'll have 51 completed cycles. If fact, they do have positive integer solutions:

$$x = 37 \quad y = 14\,. \tag{11}$$

9 Word Problem #23.4

Question 311277:[4] Albert buys and sells books, and always purchase it at the same price. He then sells the books for $5 more than what he paid for. Two months before, he broke even after buying 56 books and selling 49. What is his buying price and selling price?

10 Solution 23.4.1: Conceptualizing the Problem

Let P represent Albert's buying price, therefore, his selling price is $P+5$. The net income will be the gains minus the losses, as shown in the figure below.

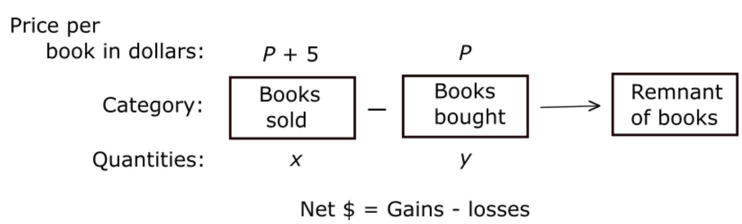

Figure 4. The net income in dollars is the sum of its part; it's just that one of those parts is negative.

To say that Albert 'broke even' means that his net for that period of time was zero. Therefore, with $x = 49$ and $y = 56$, we get

$$(P+5)\cdot 49 - P\cdot 56 = 0\,, \tag{12}$$

which has solution $P = 35$. Therefore, $P + 5 = 40$.

[4]Ibid.

11 Conclusion

By rights, I should have used a '+' sign between the rectangles in Firgure 4. But I decided that the logic behind that choice was outweighed by the possible confusion that might occurr to the novice problem solver if I put the minus sign in front of the P on top of the 'Books bought' rectangle.